Set Language and Notation

Sets, Venn Diagrams, Union and Intersection

Secondary 4 — G3 Mathematics

Targets

- Read and write sets in set language and set notation.
- Show sets in Venn diagrams.
- Find the complement of a set, and the union and the intersection of two sets.
- Write problems in set notation and solve them with the help of Venn diagrams.

Notes

1. A **set** is a collection of objects that is **well-defined**. Each object in a set is called an **element** (or a **member**) of the set.

2. We write the elements of a set between braces, { }. We usually name a set with a capital letter, such as A, B or C, and write its elements as small letters, numbers or words.

E.g. 1 The colours of a traffic light: $T = \{\text{red}, \text{amber}, \text{green}\}$.

E.g. 2 The even numbers between 1 and 9: $E = \{2, 4, 6, 8\}$.

E.g. 3 The months of the year with exactly 30 days: $M = \{\text{April, June, September, November}\}$.

3. A set can be described in three ways: by **listing** its elements, **in words**, or in **set notation**.

Listing	**In words**	**Set notation**
$A = \{1, 4, 9, 16\}$	A is the set of perfect squares less than 20.	$A = \{x :$ x is a perfect square less than 20$\}$
$B = \{l, e, v\}$	B is the set of letters in the word 'level'.	$B = \{x :$ x is a letter in the word 'level'$\}$
$C = \{2, 4, 6, 8, \dots\}$	C is the set of positive even numbers.	$C = \{x :$ x is a positive even number$\}$

- The three dots "…" stand for elements that are left out. At the end of a list, as in C, they show that the list carries on without end. Between two elements, as in $\{1, 2, 3, \dots, 20\}$, they stand for the numbers in between.
- $\{x : x \text{ is a positive even number}\}$ is read as "the set of all x such that x is a positive even number".

4. If x **belongs to** set A, we write $x \in A$. If x **does not belong to** set A, we write $x \notin A$.

E.g. 1 $A = \{3, 6, 9, 12\}$. Then $6 \in A$ and $12 \in A$, but $8 \notin A$.

E.g. 2 $B = \{x : x \text{ is a root of } (x + 2)(x - 4) = 0\}$.
$(x + 2)(x - 4) = 0$ gives $x + 2 = 0$ or $x - 4 = 0$, so $x = -2$ or $x = 4$.
$\therefore B = \{-2, 4\}$. So $-2 \in B$ and $4 \in B$, but $2 \notin B$.

5. A set must be **well-defined**: for any object, we must be able to decide for certain whether or not it belongs to the set.

E.g. 1 The set of odd numbers less than 100 is well-defined. We can check any number and say whether it belongs.

E.g. 2 The set of tall students in a class is *not* well-defined. The word 'tall' has no exact meaning.

6. The **order** of the elements does not matter, and **each element is written only once**.
E.g. For $B = \{\text{letters in the word 'seen'}\}$, we may write $B = \{s, e, n\}$ or $\{n, e, s\}$ or $\{e, s, n\}$.

7. A set with no elements is called the **empty set** (or **null set**). It is written $\varnothing$ or { }.

E.g. $P = \{\text{squares with five sides}\}$, so $P = \varnothing$.
$Q = \{\text{days of the week starting with the letter K}\}$, so $Q = \varnothing$.

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$R = \{x : x \text{ is a real number and } x^2 = -4\}$, so $R = \varnothing$ [*the square of a real number is never negative*].

> **Tip.** Never write the empty set as $\{\varnothing\}$. The set $\{\varnothing\}$ has *one* element, namely $\varnothing$, so it is not empty. In the same way, $\{0\}$ has one element, the number 0, and is not the empty set.

8. $n(A)$ stands for the **number of elements** in set A.

E.g. 1 $A = \{2, 4, 6, 8, 10\}$, so $n(A) = 5$.

E.g. 2 $B = \{\text{letters in the word 'banana'}\} = \{b, a, n\}$, so $n(B) = 3$ [*a repeated letter is counted once*].

E.g. 3 $C = \{x : x \text{ is a positive even number}\} = \{2, 4, 6, \dots\}$. The list never ends, so $n(C)$ is infinite.

E.g. 4 $D = \{x : x \text{ is an integer and } 2x = 7\}$. Solving, $x = \frac{7}{2} = 3\frac{1}{2}$, which is not an integer.
$\therefore D = \varnothing$ and $n(D) = 0$.

> **Tips: describing and listing sets.**
>
> - A set can often be described in set notation in more than one way. Any description is correct as long as it gives exactly the same elements.
> - When the elements go up in equal steps, look at the remainder that each element leaves when it is divided by the size of the step.
> - For a set given by a rule such as $x = 3k + 2$, substitute each allowed value of k in turn. If a value comes up more than once, list it only once.

9. Two sets are **equal** if they have exactly the same elements. We then write $A = B$. If two sets are not equal, we write $A \neq B$.

E.g. 1 $A = \{5, 6, 7\}$ and $B = \{7, 5, 6\}$. So $A = B$.

E.g. 2 $C = \{\text{letters in the word 'listen'}\} = \{l, i, s, t, e, n\}$ and $D = \{\text{letters in the word 'silent'}\} = \{s, i, l, e, n, t\}$.
So $C = D$ [*the order of the elements does not matter*].

E.g. 3 $E = \{x : 2x + 1 = 9\}$ and $F = \{x : x^2 = 16\}$.
$2x + 1 = 9$ gives $x = 4$, so $E = \{4\}$. $x^2 = 16$ gives $x = \pm\sqrt{16} = 4$ or -4, so $F = \{4, -4\}$.
So $E \neq F$.

E.g. 4 $G = \{d, o, g, dog\}$ and $H = \{g, o, d\}$. So $G \neq H$: G has an extra element, the word *dog*. Its letters are in H, but the word itself is not.

10. If every element of set A is also an element of set B, then A is a **subset** of B. We write $A \subseteq B$ (or $B \supseteq A$).
If A is **not a subset** of B, we write $A \not\subseteq B$.

E.g. 1 $A = \{4, 5\}$ and $B = \{3, 4, 5\}$.
$A \subseteq B$, since 4 and 5 are both in B. $B \not\subseteq A$, since $3 \in B$ but $3 \notin A$.

E.g. 2 $A = \{x, y, z\}$ and $B = \{z, x, y\}$.
$A \subseteq B$ and $B \subseteq A$, since every element of each set is in the other.

> **Tip.** If $A \subseteq B$ and $B \subseteq A$, the two sets have exactly the same elements, so $A = B$.

11. If $A \subseteq B$ and B has **at least one element that is not in** A, then A is a **proper subset** of B. We write $A \subset B$. If A is not a proper subset of B, we write $A \not\subset B$.

E.g. $A = \{m, n\}$, $B = \{n, m\}$ and $C = \{k, m, n, p\}$.
$A \subset C$: every element of A is in C, and C has elements (k and p) that are not in A. In the same way, $B \subset C$.
$A \subseteq B$ and $B \subseteq A$, so $A = B$. So A is a subset of B but *not* a proper subset: $A \not\subset B$.

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$\subseteq$	is a subset of
$\not\subseteq$	is not a subset of
$\subset$	is a proper subset of
$\not\subset$	is not a proper subset of

Tip. The empty set is a subset of every set: for any set A, $\varnothing \subseteq A \subseteq \xi$, where ξ is the universal set (note 12).

Listing subsets. List the subsets by size: the empty set first, then the subsets with one element, then those with two elements, and so on, up to the set itself. Remember $\varnothing$ and the set itself. Every subset except the set itself is a proper subset.

Tip: do not mix up elements and subsets.

- $2 \in \{2, 4, 6\}$: the number 2 is an element of $\{2, 4, 6\}$. But $\{2\} \notin \{2, 4, 6\}$.
- $\{2\} \subset \{2, 4, 6\}$: the set $\{2\}$ is a proper subset of $\{2, 4, 6\}$. We cannot write $2 \subset \{2, 4, 6\}$, because 2 is a number, not a set.
- $\{q\} \in \{p, \{q\}, r\}$: here the set $\{q\}$ is itself one of the elements.

12. The **universal set**, written ξ, is the set of all the elements that a question deals with.

E.g. 1 $A =$ {letters in the word 'cold'} $= \{c, o, l, d\}$ and $B =$ {letters in the word 'warm'} $= \{w, a, r, m\}$.
The universal set could be the set of the 26 letters of the English alphabet: $\xi = \{a, b, c, \ldots, z\}$.

E.g. 2 For $A = \{\text{football}, \text{tennis}, \text{swimming}\}$, the universal set could be $\xi = \{\text{all sports}\}$.

E.g. 3 For $B = \{5, 10, 15, 20\}$, the universal set could be $\xi = \{x : x \text{ is a multiple of } 5\}$.

13. A **Venn diagram** uses shapes to show sets.

- A large rectangle stands for the universal set, ξ.
- Circles or ovals drawn inside the rectangle stand for subsets of ξ.

Drawing a Venn diagram from lists of elements.

1. Write the elements that are in both sets in the part where the two circles overlap.
2. Write the other elements of each set in the rest of its own circle.
3. Write the elements that are in neither set outside the circles, inside the rectangle.

If two sets have no elements in common, draw circles that do not overlap. If every element of P is also in Q, draw the circle P inside the circle Q.

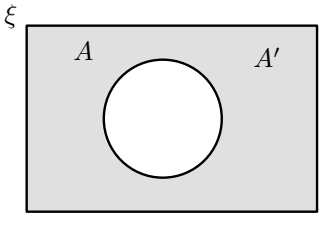
14. Two sets with **no elements in common** are called **disjoint sets**.

E.g. 1 $A = \{2, 4, 6\}$ and $B = \{1, 3, 5, 7\}$ are disjoint sets.

E.g. 2 $H =$ {months of the year starting with the letter A} $=$ {April, August} and $K =$ {months of the year starting with the letter D} $=$ {December} are disjoint sets.

15. The **complement** of set A, written A', is the set of all the elements of ξ that are **not** in A. We read A' as "A complement" or "A prime".

$$A' = \{x : x \in \xi \text{ and } x \notin A\}$$



The shaded region is A'.

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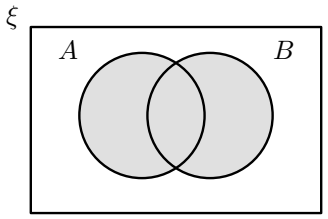
E.g. $\xi = \{2, 5, 8, 11, 14\}$ and $A = \{5, 11\}$. Then $A' = \{2, 8, 14\}$ [*the elements of ξ that are not in A*].

Tip. Two disjoint sets are not always each other's complements. $A' = B$ only when every element of ξ is in A or in B; elements that are in neither set belong to both A' and B'.

16. The **union** of two sets A and B, written $A \cup B$, is the set of the elements that are in A, **or** in B, **or** in both.

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

$\cup$ is read as "the union of".



The shaded region is $A \cup B$.

How to shade a region made from two sets. Hatch the first set one way (⟋) and the second set the other way (⟍). Then

- for a **union** ($\cup$), shade every part that carries **any** hatching;
- for an **intersection** ($\cap$), shade only the parts that carry **both** hatchings.

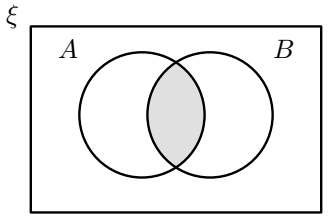
For a complement of a bracket, such as $(A \cup B)'$, first find the region inside the bracket; the answer is everything else.

Tip. A set such as $A' \cup B'$ can be read off the Venn diagram, or found by listing A' and B' first and then joining the two lists.

17. The **intersection** of two sets A and B, written $A \cap B$, is the set of the elements that are in **both** A **and** B.

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

$\cap$ is read as "the intersection of".



The shaded region is $A \cap B$.

Notes on intersections.

- Two sets A and B are disjoint exactly when $A \cap B = \varnothing$.
- If every element of A is in B, then $A \cap B = A$.
- A number that is a multiple of both a and b is a multiple of their lowest common multiple, which is not always $a \times b$.

Drawing a set from given conditions. Turn each condition into a picture. If $C \subset X$, draw C inside the region X. If $X \cap C = \varnothing$, draw C so that it does not meet X.

Finding the largest or smallest value of k. When $\xi = \{1, 2, 3, \ldots, k\}$ and you are told how many elements a set has:

1. List the numbers that could be in the set, in order.

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2. If the set has n elements, the nth number in the list must be in ξ, but the next number must not be.
3. This gives the smallest and the largest possible values of k.

18. Summary of set language and notation

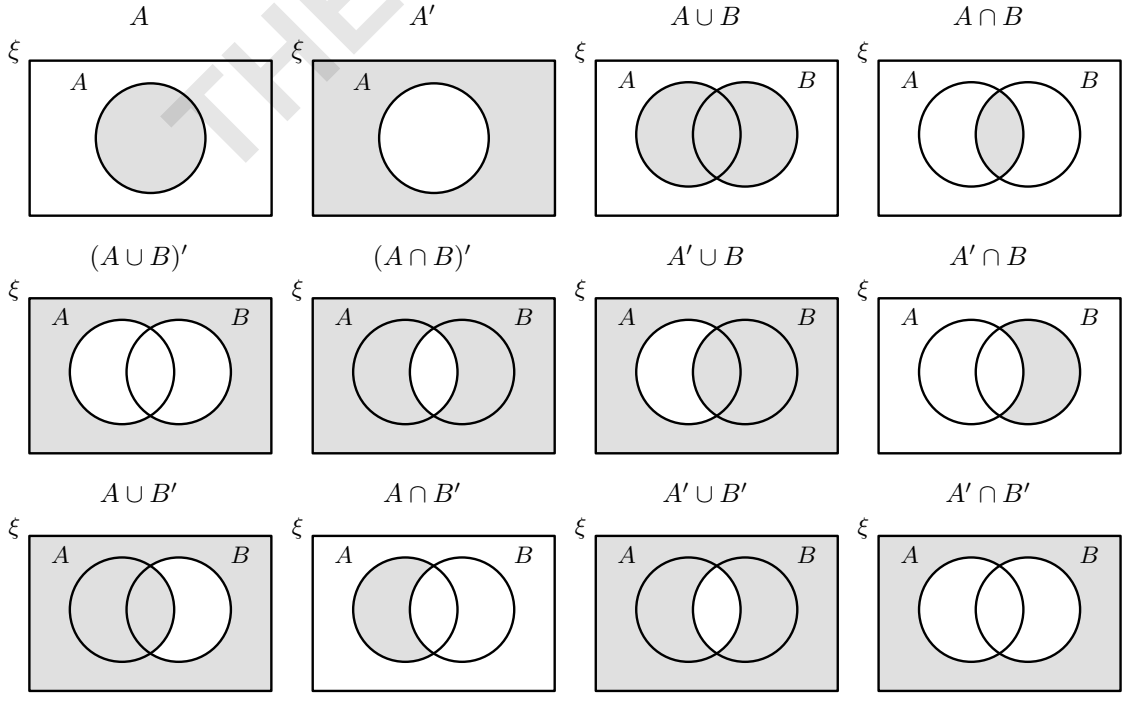
$\in$	is an element of	$n(A)$	the number of elements in set A
$\notin$	is not an element of	ξ	the universal set
$\subseteq$	is a subset of	$\varnothing$ or $\{\ \}$	the empty set (null set)
$\not\subseteq$	is not a subset of	A'	the complement of set A
$\subset$	is a proper subset of	$A \cup B$	the union of A and B
$\not\subset$	is not a proper subset of	$A \cap B$	the intersection of A and B

19. Some sets of numbers that often appear in questions on sets.

Integers $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$
Negative integers $-1, -2, -3, -4, -5, \ldots$
Positive integers $1, 2, 3, 4, 5, \ldots$
Whole numbers $0, 1, 2, 3, 4, 5, \ldots$
Natural numbers $1, 2, 3, 4, 5, \ldots$
Perfect squares $1, 4, 9, 16, 25, 36, 49, \ldots$, that is, $1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2, \ldots$
Cube numbers $1, 8, 27, 64, 125, \ldots$, that is, $1^3, 2^3, 3^3, 4^3, 5^3, \ldots$
Prime numbers $2, 3, 5, 7, 11, 13, 17, 19, 23, \ldots$
Composite numbers $4, 6, 8, 9, 10, 12, \ldots$

- A **prime number** is a natural number that has exactly two different factors: 1 and itself. So 1 is *not* a prime number.
- A **composite number** is a natural number that has more than two different factors. The number 1 is neither prime nor composite.
- A **rational number** is a number that can be written as $\frac{a}{b}$, where a and b are integers and $b \neq 0$. For example, $-2\frac{1}{4}$, $-\frac{4}{7}$, 6 and $3\frac{2}{5}$ are rational numbers.

20. The shaded regions below represent the sets named above them.



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Exercises

1. For each set A below, (i) list its elements, (ii) find $n(A)$.

(a) $A = \{\text{prime numbers between 20 and 50}\}$

(b) $A = \{\text{integers } x : -4 < x \leq 2\}$

(c) $A = \{x : x \text{ is divisible by both 2 and 3, and } 0 < x < 25\}$

(d) $A = \{\text{perfect squares between 30 and 110}\}$

(e) $A = \{\text{cube numbers between 10 and 250}\}$

(f) $A = \{x : x \text{ leaves a remainder of 3 when divided by 8, and } 0 < x < 40\}$

(g) $A = \{x : x = 3k - 1, \ k \text{ is an integer and } 0 \leq k \leq 4\}$

(h) $A = \{x : x \text{ is a root of the equation } (x - 3)(2x + 1) = 0\}$

(i) $A = \{x : x \text{ is a natural number and } 2x + 3 < 12\}$

(j) $A = \{x : x \text{ is a root of the equation } x^2 = 4x\}$

(k) $A = \{x : x = ab, \text{ where } a \text{ and } b \text{ are integers, } 1 \leq a \leq 3 \text{ and } 2 \leq b \leq 4\}$

2. Describe each set below in set notation.

(a) $F = \{2, 4, 6, 8, 10, 12\}$ (b) $G = \{a, e, i, o, u\}$ (c) $H = \{\text{Saturday, Sunday}\}$

3. Put a tick (✓) beside each set that is well-defined, and a cross (✗) beside each set that is not.

(a) The set of even numbers between 51 and 99.

(b) The set of good singers in Singapore.

(c) The set of factors of 36.

4. Put a tick (✓) beside each pair of equal sets, and a cross (✗) beside each pair that are not equal.

(a) $A = \{\text{letters in the word 'cheese'}\}$
$B = \{\text{letters in the word 'chess'}\}$

(b) $A = \{\text{letters in the word 'tablet'}\}$
$B = \{\text{letters in the word 'bottle'}\}$

(c) $A = \{1, 3\} \cup \{2, 4, 6\}$
$B = \{x : x \text{ is a factor of 12 and } x < 7\}$

5. Write $\in$ or $\notin$ in each box so that the statement is true.

(a) $0 \square \{x : x \text{ is a positive integer}\}$ (b) $\{3\} \square \{x : x \text{ is a factor of 12}\}$

6. Write $\subseteq$ or $\not\subseteq$ in each box so that the statement is true.

(a) $\{2, 3\} \square \{1, 3, 5, 7\}$ (b) $\{x : x^2 = 49\} \square \{-7, 0, 7\}$

7. $B = \{\text{letters in the word 'committee'}\}$. Each statement below is false. Change one symbol in each statement so that it becomes true.

(a) $\{c, o, m, i, t, e\} \subset B$ (b) $s \in B$ (c) $\{m, e\} \not\subseteq B$

8. Decide whether each statement is True (T) or False (F).

(a) $\{\text{letters in the word 'pan'}\} = \{n, a, p\}$ (b) $9 \in \{x : x \text{ is a prime number}\}$

(c) $-2 \in \{x : x \text{ is a root of } (x + 2)(x - 3) = 0\}$ (d) $\{a, b\} \in \{a, b, \{a, b\}\}$

9. Write down every subset of each set.

(a) $\{\text{vowels in the word 'queue'}\}$

(b) $\{x : x \text{ is a prime number less than 6}\}$

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10. $A = \{p, q, r, s\}$.

(a) List all the subsets of A that have exactly two elements.

(b) List all the subsets of A that have exactly three elements.

(c) Find the total number of subsets of A.

(d) Write down the number of proper subsets of A.

11. ξ = {football, badminton, tennis, swimming, cycling, netball}, A = {football, netball, tennis, badminton} and B = {badminton, tennis}.

(a) Show the sets in a Venn diagram.

(b) List the elements of (i) A', (ii) B'.

(c) State the relation between A and B.

12. $\xi = \{x : x \text{ is a positive integer less than } 11\}$. For each pair of sets below, (i) show A and B in a Venn diagram, (ii) list the elements of $A \cup B$ and of $A \cap B$.

(a) $A = \{2, 3, 4, 5, 6\}$, $B = \{1, 3, 5, 7, 9\}$

(b) $A = \{\text{multiples of } 3\}$, $B = \{\text{factors of } 12\}$

13. For each pair of sets, list the elements of $A \cup B$ and of $A \cap B$.

(a) $A = \{12, 13, 14, 15\}$, $B = \{14, 16, 18\}$ (b) $A = \{p, q, r, s, t\}$, $B = \{r, t, u, v\}$

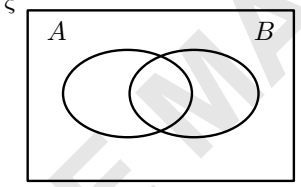
(c) $A = \{\text{factors of } 12\}$, $B = \{\text{factors of } 8\}$

14. Given that $A = \{5, 10, 15, 20\}$, $B = \{4, 8, 12, 16, 20\}$ and $C = \{10, 12, 16, 20\}$, find

(a) $n(A \cap C)$, (b) $n(B \cap C)$, (c) $n(A \cap B \cap C)$, (d) $n(A \cup B \cup C)$.

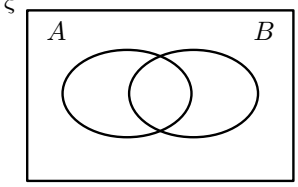
15. Shade the region named under each Venn diagram.

(a)



$(A' \cap B')'$

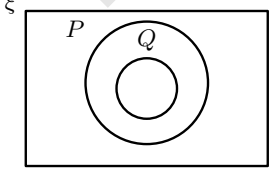
(b)



$(A \cup B)' \cup (A \cap B)$

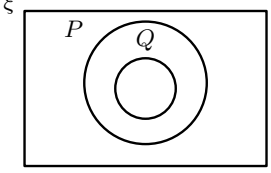
16. In each Venn diagram, $Q \subset P$. Shade the region named under each diagram.

(a)



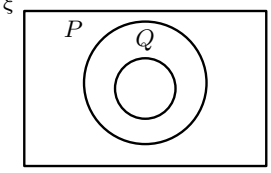
$(P \cap Q') \cup Q$

(b)



$(P' \cup Q') \cap P$

(c)



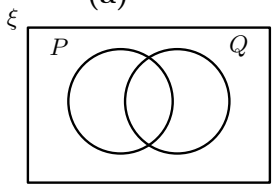
$Q \cap P'$

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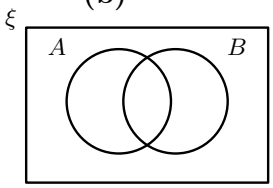
17. On each Venn diagram, shade the region represented by the set named under it. In (c), $B \subset A$.

(a)



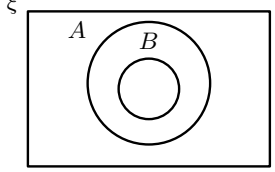
$(P \cap Q) \cup (P \cap Q')$

(b)



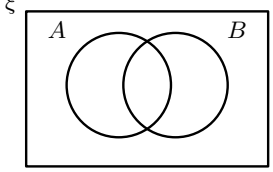
$(A \cap B')' \cap A$

(c)



$(A \cup B') \cap B$

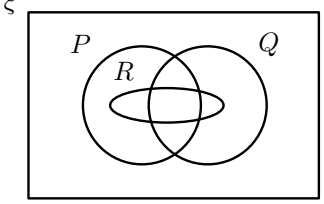
(d)



$(A \cup B) \cap (A' \cup B)$

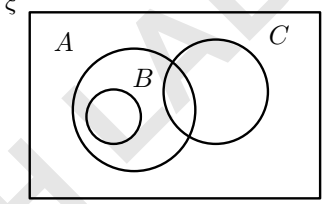
18. Shade the region that represents each set.

(a)



$(P \cup Q) \cap R'$

(b)



$(A \cap C) \cup B$

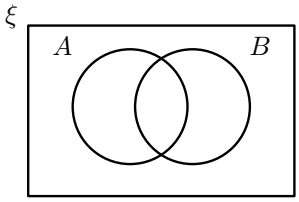
19. $\xi = \{\text{integers } x : 1 \leq x \leq 14\}$, $P = \{\text{perfect squares}\}$ and $Q = \{\text{odd numbers}\}$.

(a) Draw a Venn diagram to show this information.

(b) List the elements of $P' \cap Q$.

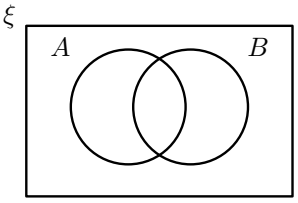
(c) Find $n(P \cup Q)$.

20. (a) Shade the set $(A \cup B) \cap A'$.



(b) $\xi = \{x : x \text{ is an integer and } 10 < x \leq 60\}$, $M = \{x : x \text{ is a prime number}\}$ and $N = \{x : \text{the digits of } x \text{ add up to } 8\}$.
Find $n(M \cap N)$.

21. The Venn diagram shows the sets ξ, A and B.
Add a set C to the Venn diagram so that $C \subset B$, $A \cap C \neq \varnothing$ and $A' \cap C \neq \varnothing$.



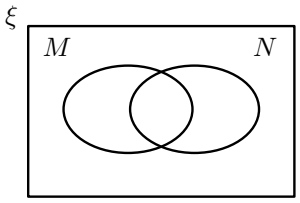
22. $\xi = \{\text{integers } x : -6 \leq x \leq 6\}$, $A = \{x : -2 < x \leq 6\}$ and $B = \{x : x^2 \leq 9\}$.
List the elements of (a) A', (b) $A \cap B$, (c) $A \cup B$.

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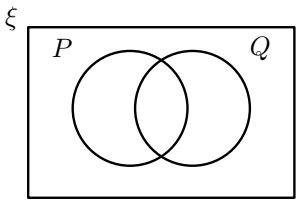
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23. (a) $\xi = \{\text{integers } x : 1 \leq x \leq 12\}$, $M = \{\text{multiples of 3}\}$ and $N = \{\text{even numbers}\}$.
List the elements of $M \cap (M \cap N)'$.

(b) In the Venn diagram, shade the region which represents $M \cap (M \cap N)'$.



24. (a) In the Venn diagram, shade the set represented by $P \cup (P' \cap Q)$.



(b) $\xi = \{x : x \text{ is an integer and } 20 \leq x \leq 35\}$, $S = \{x : x \text{ is a multiple of } y\}$ and $T = \{x : x \text{ is a prime number}\}$, where y is a positive integer.

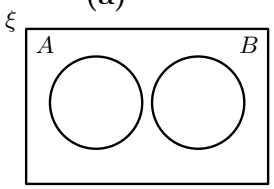
(i) List the elements of T.

(ii) Find the smallest possible value of y such that $n(S \cap T) = 0$.

(iii) Find all the possible values of y such that $n(S \cap T) = 1$.

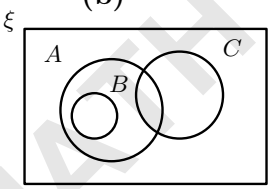
25. On the Venn diagrams, shade the regions which represent the sets named under them.

(a)



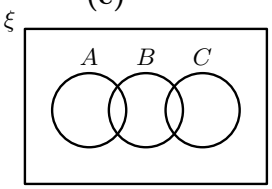
$(A \cup B)' \cup (A \cap B)$

(b)



$A \cap C'$

(c)

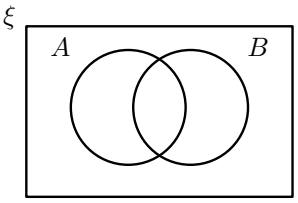


$B \cap (A \cup C)$

(In (a), A and B are disjoint. In (b), $B \subset A$, and C does not meet B. In (c), A and C do not meet.)

26. The universal set ξ contains three sets P, Q and R such that $P \subset R$, $Q \subset R$ and $P \cap Q = \varnothing$.
Show these sets in a Venn diagram.

27. $\xi = \{\text{all triangles}\}$, $A = \{\text{right-angled triangles}\}$ and $B = \{\text{isosceles triangles}\}$.
P is a triangle with angles 25°, 65° and 90°.
Q is a triangle with sides 6 cm, 7 cm and 8 cm.
R is a triangle with two angles of 45°.
S is a triangle with sides 8 cm, 15 cm and 17 cm.
Write P, Q, R and S in the correct regions of the Venn diagram.



28. $\xi = \{x : x \text{ is a natural number}\}$, $A = \{x : x \text{ is a perfect square}\}$ and $B = \{x : x \text{ is a prime number}\}$.

(a) List the elements of A and of B.

(b) Find (i) $A \cap B$, (ii) $n(A \cap B)$.

(c) Draw a Venn diagram to represent the sets ξ, A and B, and write the numbers 1, 4, 7 and 10 in the correct regions.

29. $A = \{2, 4, 6, 8, 10, 12\}$ and $B = \{3, 6, 9, 12, 15\}$.

(a) Find (i) $A \cap B$, (ii) $A \cup B$, (iii) $n(A \cap B)$, (iv) $n(A \cup B)$.

(b) Explain why $n(A \cup B)$ is not equal to $n(A) + n(B)$.

(c) Write down a set C such that $n(C) = 2$, $C \subset A$ and $C \cap B = \varnothing$.

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30. $\xi = \{11, 12, 13, 14, 15, 16, 17\}$, $A = \{11, 12, 14, 16\}$ and $B = \{12, 13, 16, 17\}$.

(a) Find (i) $(A \cup B)'$, (ii) $(A \cap B)'$, (iii) $A' \cup B'$, (iv) $A' \cap B'$.

(b) Use your answers to (a) to fill in each box with $\cup$ or $\cap$.

$(A \cup B)' = A' \square B'$ $(A \cap B)' = A' \square B'$

31. $\xi = \{b, c, d, e, f, g, h\}$. The sets M and N are such that $M \cap N = \{d\}$, $M \cap N' = \{b, f\}$ and $M' \cap N = \{c, e\}$.

(a) Draw a Venn diagram to show ξ, M and N.

(b) List the elements of (i) M, (ii) N, (iii) $(M \cup N)'$.

(c) Find $n(M' \cup N')$.

32. $\xi = \{\text{square, rectangle, rhombus, parallelogram, kite}\}$.
$P = \{x : \text{the diagonals of } x \text{ cut each other at right angles}\}$,
$Q = \{x : \text{the diagonals of } x \text{ are equal in length}\}$ and
$R = \{x : \text{both pairs of opposite sides of } x \text{ are parallel}\}$.

(a) List the elements of (i) P, (ii) Q, (iii) R.

(b) Find (i) $P \cap Q$, (ii) $P \cap R$.

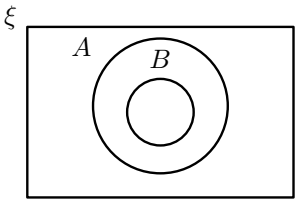
(c) Describe the relation between Q and R.

(d) Name the element of P that is not in R.

33. $\xi = \{\text{rational numbers}\}$, $A = \{\text{integers}\}$, $B = \{\text{whole numbers}\}$, $C = \{\text{natural numbers}\}$ and $D = \{\text{rational numbers}\}$.
Write each of the following as simply as possible.

(a) $B \cup C$ (b) $A \cup D$ (c) $B \cap C$ (d) $A \cap D$ (e) $B \cap C'$

34. Use the Venn diagram to state whether each statement is True (T) or False (F).



(a) $A' \subset B'$ (b) $A \cap B = B$

(c) $A \cup B = B$ (d) $(A \cup B)' = A'$

35. Two sets P and Q are such that $P \subset Q \subset \xi$.

(a) Draw a Venn diagram to show ξ, P and Q.

(b) Write down, as simply as possible, (i) $P \cap Q$, (ii) $P \cup Q$, (iii) $P' \cup Q$, (iv) $P \cap Q'$.

36. Is each statement True (T) or False (F)?

(a) If $A = \{\text{prime numbers}\}$ and $B = \{\text{odd numbers}\}$, then $A \subset B$.

(b) If $A \subset B$, then $A \neq B$.

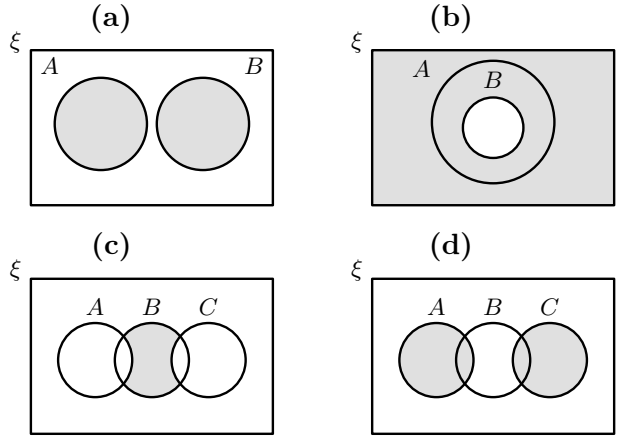
(c) If $A \cup B = \varnothing$, then $A = \varnothing$ and $B = \varnothing$.

(d) If $A \cup B = A$, then $B \subseteq A$.

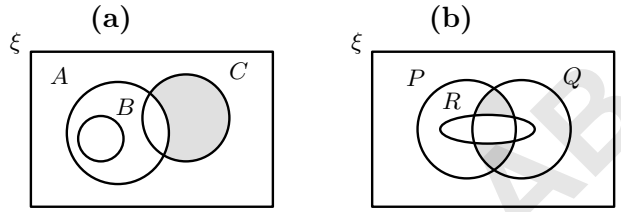
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37. Describe the shaded region in each Venn diagram in set notation.

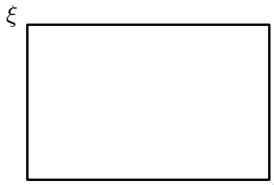


38. Describe the shaded region in each Venn diagram in set notation, using the sets shown.



39. A and B are subsets of the universal set ξ. Complete each Venn diagram so that it shows the conditions given above it.

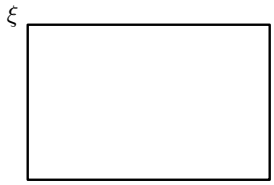
(a) $A \cup B = A$, $A \neq B$



(b) $A \cap B' = \varnothing$, $A \neq B$

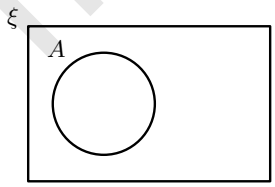


(c) $A \cap B \neq \varnothing$, $A \not\subseteq B$, $B \not\subseteq A$

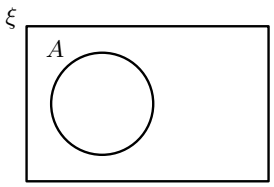


40. A, B and C are subsets of the universal set ξ, and no two of them are equal. Complete each Venn diagram so that it shows the conditions above it.

(a) $A \cap B = B$ and $B \cup C = B$



(b) $B \cap C' = \varnothing$ and $A \cap C = \varnothing$



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Solutions to Exercises

List the elements of each set first; then find each complement, union or intersection from those lists. To shade a region, hatch the two sets it is made from, and shade every part with **any** hatching for a union, or only the parts with **both** hatchings for an intersection. Final answers are boxed.

1(a) $\boxed{A = \{23, 29, 31, 37, 41, 43, 47\}}$, $\boxed{n(A) = 7}$

1(b) $\boxed{A = \{-3, -2, -1, 0, 1, 2\}}$, $\boxed{n(A) = 6}$ [-4 is not included, but 2 is]

1(c) A number divisible by both 2 and 3 is a multiple of 6.
$\boxed{A = \{6, 12, 18, 24\}}$, $\boxed{n(A) = 4}$

1(d) $6^2, 7^2, 8^2, 9^2, 10^2$: $\boxed{A = \{36, 49, 64, 81, 100\}}$, $\boxed{n(A) = 5}$

1(e) $3^3, 4^3, 5^3, 6^3$: $\boxed{A = \{27, 64, 125, 216\}}$, $\boxed{n(A) = 4}$

1(f) Add 3 to the multiples of 8 $(0, 8, 16, 24, 32)$:
$0 + 3 = 3$, $8 + 3 = 11$, $16 + 3 = 19$, $24 + 3 = 27$, $32 + 3 = 35$ [the next, 43, is too big]
$\boxed{A = \{3, 11, 19, 27, 35\}}$, $\boxed{n(A) = 5}$ [$3 = 0 \times 8 + 3$, so 3 is in A]

1(g) $k = 0, 1, 2, 3, 4$:
$A = \{3(0) - 1,\ 3(1) - 1,\ 3(2) - 1,\ 3(3) - 1,\ 3(4) - 1\}$
$\boxed{A = \{-1, 2, 5, 8, 11\}}$, $\boxed{n(A) = 5}$

1(h) $(x - 3)(2x + 1) = 0$; $x - 3 = 0$ or $2x + 1 = 0$;
$x = 3$ or $x = -\frac{1}{2}$
$\boxed{A = \{-\frac{1}{2}, 3\}}$, $\boxed{n(A) = 2}$

1(i) $2x + 3 < 12$; $2x < 9$; $x < 4\frac{1}{2}$
$x = 1, 2, 3, 4$ [x is a natural number]
$\boxed{A = \{1, 2, 3, 4\}}$, $\boxed{n(A) = 4}$

1(j) $x^2 = 4x$; $x^2 - 4x = 0$; $x(x - 4) = 0$; $x = 0$ or $x = 4$
$\boxed{A = \{0, 4\}}$, $\boxed{n(A) = 2}$ [do not divide by x: that loses $x = 0$]

1(k) $a = 1, 2, 3$ and $b = 2, 3, 4$:

$a \backslash b$	2	3	4
1	$1 \times 2 = 2$	$1 \times 3 = 3$	$1 \times 4 = 4$
2	$2 \times 2 = 4$	$2 \times 3 = 6$	$2 \times 4 = 8$
3	$3 \times 2 = 6$	$3 \times 3 = 9$	$3 \times 4 = 12$

$\boxed{A = \{2, 3, 4, 6, 8, 9, 12\}}$, $\boxed{n(A) = 7}$ [4 and 6 are listed once]

2 Other answers are possible.

(a) $\boxed{F = \{x : x \text{ is a positive even number and } x \leq 12\}}$

(b) $\boxed{G = \{x : x \text{ is a vowel in the English alphabet}\}}$

(c) $\boxed{H = \{x : x \text{ is a day of the week starting with the letter S}\}}$

3(a) $\boxed{\checkmark}$ Well-defined: we can check whether any number is an even number between 51 and 99.

3(b) $\boxed{\times}$ Not well-defined: we cannot measure 'good'.

3(c) $\boxed{\checkmark}$ Well-defined: the factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18 and 36.

4(a) $A = \{c, h, e, s\}$, $B = \{c, h, e, s\}$
$\boxed{\checkmark}$, $\boxed{A = B}$ [repeated letters are listed once]

4(b) $A = \{t, a, b, l, e\}$, $B = \{b, o, t, l, e\}$
$\boxed{\times}$, $\boxed{A \neq B}$ [$a \in A$ but $a \notin B$]

4(c) $A = \{1, 3\} \cup \{2, 4, 6\} = \{1, 2, 3, 4, 6\}$
$B = \{1, 2, 3, 4, 6\}$ [the factors of 12 are 1, 2, 3, 4, 6, 12]
$\boxed{\checkmark}$, $\boxed{A = B}$

5 (a) $0\ \boxed{\notin}\ \{1, 2, 3, \dots\}$
(b) $\{3\}\ \boxed{\notin}\ \{1, 2, 3, 4, 6, 12\}$
[in (a), 0 is not a positive integer; in (b), $\{3\}$ is a subset of $\{1, 2, 3, 4, 6, 12\}$, not an element]

6 (a) $\{2, 3\}\ \boxed{\not\subseteq}\ \{1, 3, 5, 7\}$ [$2 \in \{2, 3\}$ but $2 \notin \{1, 3, 5, 7\}$]
(b) $x^2 = 49$ gives $x = \pm 7$, so the set is $\{-7, 7\}$.
$\{-7, 7\}\ \boxed{\subseteq}\ \{-7, 0, 7\}$

7 $B = \{c, o, m, i, t, e\}$
(a) $\boxed{\{c, o, m, i, t, e\} \subseteq B}$ [or $\{c, o, m, i, t, e\} = B$; the set is B itself, so it is not a proper subset]
(b) $\boxed{s \notin B}$
(c) $\boxed{\{m, e\} \subseteq B}$ [or $\{m, e\} \subset B$]

8 (a) $\boxed{\text{T}}$, both sets are $\{p, a, n\}$ [the order does not matter]
(b) $\boxed{\text{F}}$, $9 = 3 \times 3$ is not a prime number
(c) $\boxed{\text{T}}$, the roots are -2 and 3
(d) $\boxed{\text{T}}$ [the set $\{a, b\}$ is itself one of the three elements]

9 (a) The vowels in 'queue' are u and e.
$\boxed{\varnothing,\ \{u\},\ \{e\},\ \{u, e\}}$
(b) The set is $\{2, 3, 5\}$.
$\boxed{\varnothing,\ \{2\},\ \{3\},\ \{5\},\ \{2, 3\},\ \{2, 5\},\ \{3, 5\},\ \{2, 3, 5\}}$

10(a) $\boxed{\{p, q\},\ \{p, r\},\ \{p, s\},\ \{q, r\},\ \{q, s\},\ \{r, s\}}$

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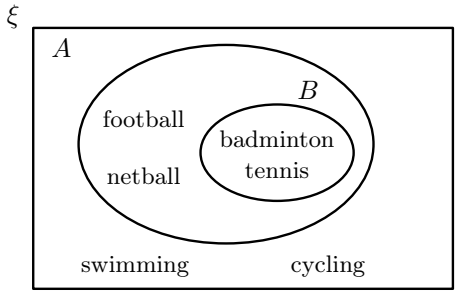
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10(b) $\boxed{\{p,q,r\},\ \{p,q,s\},\ \{p,r,s\},\ \{q,r,s\}}$
[each one leaves out one element of A]

10(c) By size: 1 empty set, 4 one-element subsets, 6 two-element subsets, 4 three-element subsets and 1 four-element subset (A itself).
Number of subsets $= 1+4+6+4+1 = \boxed{16}$

10(d) Every subset except A itself is a proper subset.
Number of proper subsets $= 16-1 = \boxed{15}$

11(a) [every element of B is in A, so the oval B is drawn inside A]

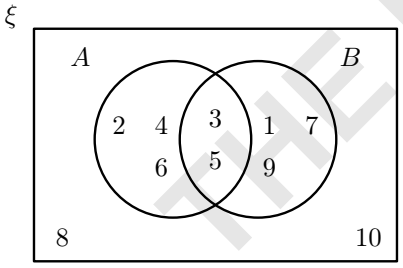


11(b) (i) $\boxed{A' = \{\text{swimming, cycling}\}}$
(ii) $\boxed{B' = \{\text{football, netball, swimming, cycling}\}}$

11(c) $\boxed{B \subset A}$ [B is a proper subset of A: A also has football and netball]

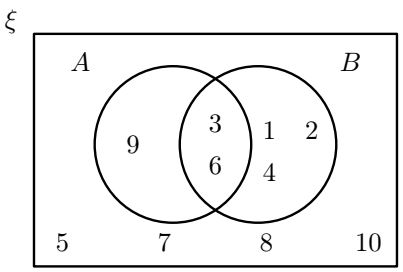
12 $\xi = \{1,2,3,4,5,6,7,8,9,10\}$

12(a) (i)



(ii) $\boxed{A \cup B = \{1,2,3,4,5,6,7,9\}}$, $\boxed{A \cap B = \{3,5\}}$

12(b) $A = \{3,6,9\}$, $B = \{1,2,3,4,6\}$ [12 is not in ξ]
(i)



(ii) $\boxed{A \cup B = \{1,2,3,4,6,9\}}$, $\boxed{A \cap B = \{3,6\}}$

13(a) $\boxed{A \cup B = \{12,13,14,15,16,18\}}$,
$\boxed{A \cap B = \{14\}}$

13(b) $\boxed{A \cup B = \{p,q,r,s,t,u,v\}}$, $\boxed{A \cap B = \{r,t\}}$

13(c) $A = \{1,2,3,4,6,12\}$, $B = \{1,2,4,8\}$
$\boxed{A \cup B = \{1,2,3,4,6,8,12\}}$, $\boxed{A \cap B = \{1,2,4\}}$

14(a) $A \cap C = \{10,20\}$; $\boxed{n(A \cap C) = 2}$

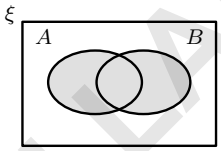
14(b) $B \cap C = \{12,16,20\}$; $\boxed{n(B \cap C) = 3}$

14(c) $A \cap B = \{20\}$ and $20 \in C$, so $A \cap B \cap C = \{20\}$;
$\boxed{n(A \cap B \cap C) = 1}$

14(d) $A \cup B \cup C = \{4,5,8,10,12,15,16,20\}$;
$\boxed{n(A \cup B \cup C) = 8}$

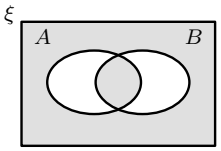
15

(a) $(A' \cap B')'$



[$A' \cap B'$ is outside both ovals, so $(A' \cap B')'$ is everything inside them: $A \cup B$]

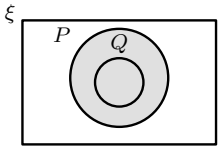
(b) $(A \cup B)' \cup (A \cap B)$



[outside both ovals, together with the overlap]

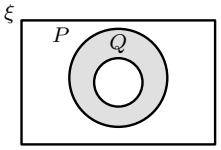
16 Q lies wholly inside P.

(a) $(P \cap Q') \cup Q$



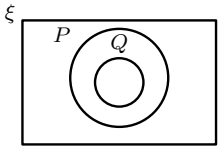
[$P \cap Q'$ is the ring; adding Q fills the whole of P]

(b) $(P' \cup Q') \cap P$



[$P' \cup Q' = Q'$, everything outside Q; its part inside P is the ring]

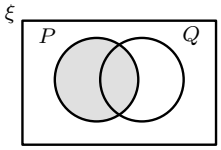
(c) $Q \cap P'$



[Q lies inside P, so no part of Q is outside P: $Q \cap P' = \varnothing$ and no region is shaded]

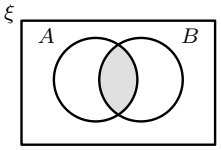
17

(a) $(P \cap Q) \cup (P \cap Q')$



[the part of P inside Q together with the part of P outside Q: all of P]

(b) $(A \cap B')' \cap A$

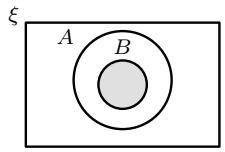


[$(A \cap B')'$ is everything except A only; its part inside A is the overlap]

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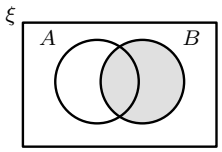
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(c) $(A \cup B') \cap B$



[B' covers everything outside B and A covers B, so $A \cup B' = \xi$; its part inside B is B]

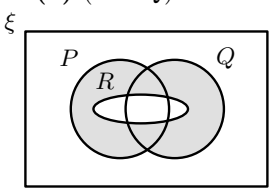
(d) $(A \cup B) \cap (A' \cup B)$



[$A' \cup B$ is everything except A only; its part inside $A \cup B$ is B]

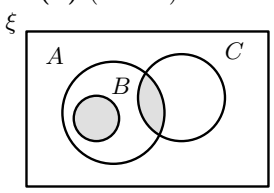
18

(a) $(P \cup Q) \cap R'$



[hatch $P \cup Q$ and R'; both hatchings: the two circles with the oval R taken out]

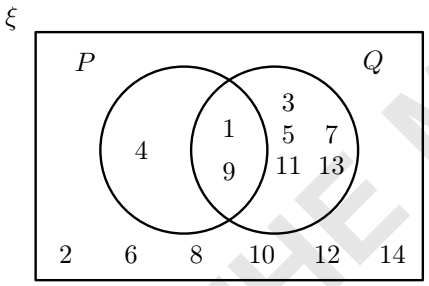
(b) $(A \cap C) \cup B$



[the overlap of A and C, together with B]

19 $\xi = \{1, 2, \ldots, 14\}$, $P = \{1, 4, 9\}$, $Q = \{1, 3, 5, 7, 9, 11, 13\}$

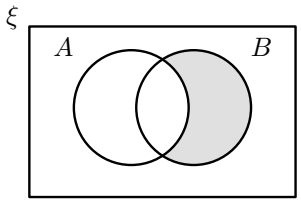
(a)



(b) $\boxed{P' \cap Q = \{3, 5, 7, 11, 13\}}$ [odd numbers that are not perfect squares]

(c) $P \cup Q = \{1, 3, 4, 5, 7, 9, 11, 13\}$; $\boxed{n(P \cup Q) = 8}$

20(a) [hatch $A \cup B$ and A'; both hatchings: the part of B outside A]



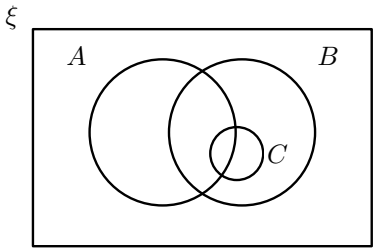
20(b) $\xi = \{11, 12, 13, \ldots, 60\}$

Digits adding up to 8: $N = \{17, 26, 35, 44, 53\}$

26 and 44 are even, and $35 = 5 \times 7$; 17 and 53 are prime.

$M \cap N = \{17, 53\}$; $\boxed{n(M \cap N) = 2}$

21 [$C \subset B$, so C lies inside B; $A \cap C \neq \varnothing$ and $A' \cap C \neq \varnothing$, so part of C is inside A and part is outside A]



22 $\xi = \{-6, -5, -4, \ldots, 4, 5, 6\}$

$A = \{-1, 0, 1, 2, 3, 4, 5, 6\}$

$x^2 \leq 9$ for $x = -3, -2, -1, 0, 1, 2, 3$ only, so $B = \{-3, -2, -1, 0, 1, 2, 3\}$

(a) $\boxed{A' = \{-6, -5, -4, -3, -2\}}$

(b) $\boxed{A \cap B = \{-1, 0, 1, 2, 3\}}$

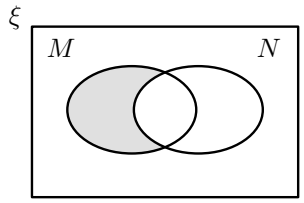
(c) $\boxed{A \cup B = \{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}}$

23(a) $\xi = \{1, 2, \ldots, 12\}$, $M = \{3, 6, 9, 12\}$, $N = \{2, 4, 6, 8, 10, 12\}$

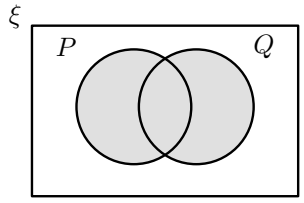
$M \cap N = \{6, 12\}$

$M \cap (M \cap N)' = \boxed{\{3, 9\}}$ [the elements of M that are not in N]

23(b) [$(M \cap N)'$ is everything except the overlap; its part inside M is the part of M outside N]



24(a) [P together with the part of Q outside P: all of $P \cup Q$]



24(b) $\xi = \{20, 21, 22, \ldots, 35\}$

(i) $\boxed{T = \{23, 29, 31\}}$

(ii) $y = 1$: $S = \xi$, which holds 23, 29 and 31.

$y = 2$: $S = \{20, 22, 24, \ldots, 34\}$. Every element is even and bigger than 2, so none is prime: $n(S \cap T) = 0$.

Smallest $y = \boxed{2}$

(iii) For $y \geq 2$, a multiple of y can be prime only if it is y itself, with y prime. So $n(S \cap T) = 1$ only when y is one of the primes in ξ (then $S = \{y\}$, since $2y > 35$).

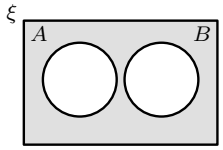
$\boxed{y = 23,\ 29 \text{ or } 31}$ [$y = 1$ gives $n(S \cap T) = 3$]

25

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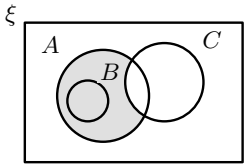
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(a) $(A \cup B)' \cup (A \cap B)$



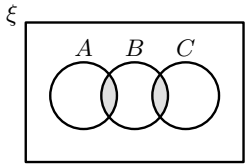
[A and B do not meet, so $A \cap B = \varnothing$ and only $(A \cup B)'$, outside both circles, is shaded]

(b) $A \cap C'$



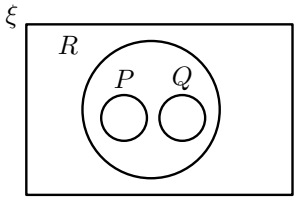
[all of A except its overlap with C; B is included]

(c) $B \cap (A \cup C)$

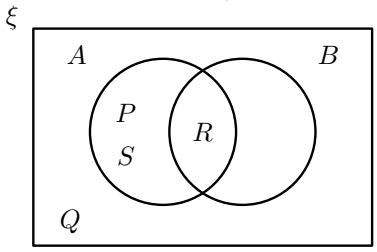


[hatch B and $A \cup C$; both hatchings: the two overlaps]

26 [$P \subset R$ and $Q \subset R$, so P and Q lie inside R; $P \cap Q = \varnothing$, so P and Q do not meet]



27 P: right-angled; its angles are all different, so it is not isosceles.
Q: $6^2 + 7^2 = 85 \neq 8^2$, so it is not right-angled; no two sides are equal, so it is not isosceles.
R: the third angle is $180° - 45° - 45° = 90°$, so it is right-angled, and it is isosceles (two equal angles).
S: $8^2 + 15^2 = 64 + 225 = 289 = 17^2$, so it is right-angled [converse of Pythagoras' theorem]; no two sides are equal, so it is not isosceles.



28(a) $\boxed{A = \{1, 4, 9, 16, 25, \dots\}}$

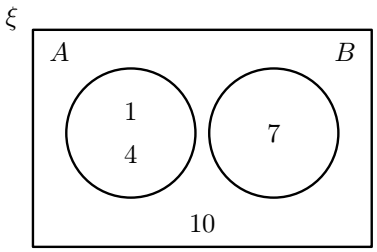
$\boxed{B = \{2, 3, 5, 7, 11, 13, \dots\}}$

28(b) (i) 1 is not a prime number. Any other perfect square $k \times k$, with $k \geq 2$, has the factor k as well as 1 and itself, so it is not prime.

$\boxed{A \cap B = \varnothing}$

(ii) $\boxed{n(A \cap B) = 0}$

28(c) [A and B are disjoint; 1 and 4 are perfect squares, 7 is prime, and 10 is neither]



29(a) (i) $\boxed{A \cap B = \{6, 12\}}$

(ii) $\boxed{A \cup B = \{2, 3, 4, 6, 8, 9, 10, 12, 15\}}$

(iii) $\boxed{n(A \cap B) = 2}$ (iv) $\boxed{n(A \cup B) = 9}$

29(b) $n(A) + n(B) = 6 + 5 = 11$, but 6 and 12 are in both sets, so they are counted twice. In $A \cup B$ each is listed only once, so $n(A \cup B) = 11 - 2 = 9$.

29(c) C must use elements of A that are not in B: 2, 4, 8 or 10.
E.g. $\boxed{C = \{2, 4\}}$ [any two of 2, 4, 8 and 10]

30(a) $A' = \{13, 15, 17\}$, $B' = \{11, 14, 15\}$

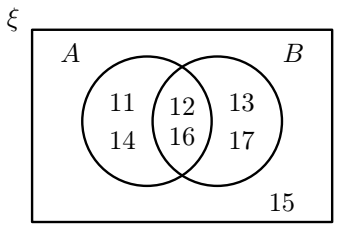
(i) $A \cup B = \{11, 12, 13, 14, 16, 17\}$; $\boxed{(A \cup B)' = \{15\}}$

(ii) $A \cap B = \{12, 16\}$; $\boxed{(A \cap B)' = \{11, 13, 14, 15, 17\}}$

(iii) $A' \cup B' = \{13, 15, 17\} \cup \{11, 14, 15\} =$
$\boxed{\{11, 13, 14, 15, 17\}}$

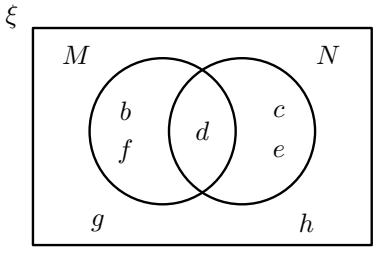
(iv) $A' \cap B' = \{13, 15, 17\} \cap \{11, 14, 15\} = \boxed{\{15\}}$

Alternative method: draw the Venn diagram and read off each region.



30(b) From (a)(i) and (iv), and from (a)(ii) and (iii):
$(A \cup B)' = A' \boxed{\cap} B'$ $(A \cap B)' = A' \boxed{\cup} B'$

31(a) [$M \cap N$ goes in the overlap, $M \cap N'$ in M only, $M' \cap N$ in N only; the rest of ξ goes outside both circles]



31(b) (i) $\boxed{M = \{b, d, f\}}$ (ii) $\boxed{N = \{c, d, e\}}$

(iii) $\boxed{(M \cup N)' = \{g, h\}}$

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31(c) $M' \cup N'$ is everything except the overlap $\{d\}$:
$M' \cup N' = \{b, c, e, f, g, h\}$; $\boxed{n(M' \cup N') = 6}$

32(a) (i) $\boxed{P = \{\text{square, rhombus, kite}\}}$

(ii) $\boxed{Q = \{\text{square, rectangle}\}}$

(iii) $\boxed{R = \{\text{square, rectangle, rhombus, parallelogram}\}}$

32(b) (i) $\boxed{P \cap Q = \{\text{square}\}}$

(ii) $\boxed{P \cap R = \{\text{square, rhombus}\}}$

32(c) $\boxed{Q \subset R}$ [Q is a proper subset of R]

32(d) $\boxed{\text{kite}}$ [the diagonals of a kite cut each other at right angles, but its opposite sides are not parallel]

33 [every natural number is a whole number; the only whole number that is not a natural number is 0, so C' holds 0 but no other whole number; every integer is a rational number]

(a) $B \cup C = \{\text{whole numbers}\} = \boxed{B}$

(b) $A \cup D = \{\text{rational numbers}\} = \boxed{D}$

(c) $B \cap C = \{\text{natural numbers}\} = \boxed{C}$

(d) $A \cap D = \{\text{integers}\} = \boxed{A}$

(e) $\boxed{B \cap C' = \{0\}}$

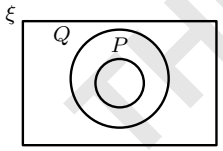
34 (a) $\boxed{\text{T}}$ [outside A is part of outside B]

(b) $\boxed{\text{T}}$ [all of B is inside A]

(c) $\boxed{\text{F}}$, $A \cup B = A$

(d) $\boxed{\text{T}}$ [$A \cup B = A$]

35(a) [$P \subset Q$, so P lies inside Q]



35(b) (i) $\boxed{P \cap Q = P}$ [all of P is inside Q]

(ii) $\boxed{P \cup Q = Q}$

(iii) P' is everything outside P; adding Q fills in P too.
$\boxed{P' \cup Q = \xi}$

(iv) No part of P is outside Q. $\boxed{P \cap Q' = \varnothing}$

36 (a) $\boxed{\text{F}}$, 2 is a prime number but not odd

(b) $\boxed{\text{T}}$ [B has an element that is not in A]

(c) $\boxed{\text{T}}$ [if either set had an element, so would $A \cup B$]

(d) $\boxed{\text{T}}$ [adding B to A adds nothing new, so every element of B is already in A]

37 Other correct descriptions are possible.

(a) $\boxed{A \cup B}$ (b) $\boxed{B'}$

(c) $\boxed{B \cap (A \cup C)'}$ or $\boxed{A' \cap B \cap C'}$

(d) $\boxed{A \cup C}$

38 Other correct descriptions are possible.

(a) The part of C outside A: $\boxed{C \cap A'}$

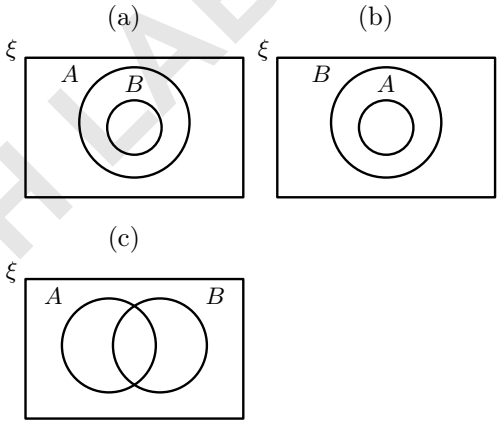
(b) The overlap of P and Q, outside the oval R:
$\boxed{P \cap Q \cap R'}$

39 [draw the two circles to match each condition]

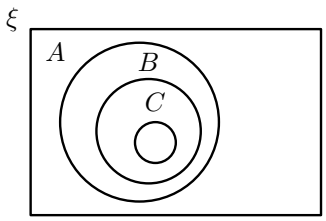
(a) $A \cup B = A$ means that B adds nothing to A: B lies inside A.

(b) $A \cap B' = \varnothing$ means that no part of A is outside B: A lies inside B.

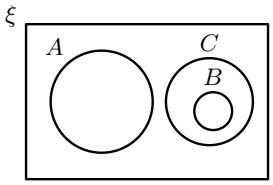
(c) The circles overlap, and neither lies inside the other.



40(a) [$A \cap B = B$, so B lies inside A; $B \cup C = B$, so C lies inside B]



40(b) [$B \cap C' = \varnothing$: no part of B is outside C, so B lies inside C; $A \cap C = \varnothing$, so C does not meet A]



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