Surds

Simplifying, Rationalising, Solving Equations and Word Problems

Secondary 3 — G3 Additional Mathematics

Notes

Rules for Surds

(a) $\sqrt{a} \times \sqrt{a} = a$

(b) $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

(c) $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$

(d) $a\sqrt{b} \times c\sqrt{d} = ac\sqrt{bd}$

(e) $m\sqrt{a} + n\sqrt{a} = (m+n)\sqrt{a}$

(f) $m\sqrt{a} - n\sqrt{a} = (m-n)\sqrt{a}$

(g) If $a + b\sqrt{k} = c + d\sqrt{k}$, then $a = c$ and $b = d$

(h) If $\sqrt{a} = b$, then $a = b^2$

(i) $(a + b\sqrt{c})$ and $(a - b\sqrt{c})$ are called **conjugates**: each is the conjugate of the other. So are $(\sqrt{a} + \sqrt{b})$ and $(\sqrt{a} - \sqrt{b})$

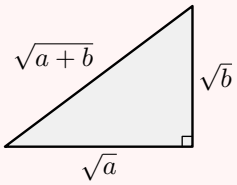
Rules (a)–(f) need the numbers under the square roots to be zero or positive, with $b \neq 0$ in (c). Rule (g) needs a, b, c and d to be rational and $\sqrt{k}$ to be irrational.

Watch out! In general,

(a) $\sqrt{a+b}$ is *not* $\sqrt{a} + \sqrt{b}$

(b) $\sqrt{a-b}$ is *not* $\sqrt{a} - \sqrt{b}$

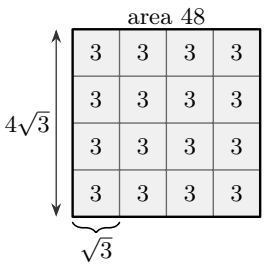
Test it with numbers: $\sqrt{36+64} = \sqrt{100} = 10$, but $\sqrt{36} + \sqrt{64} = 6 + 8 = 14$.



In a right-angled triangle whose two shorter sides are $\sqrt{a}$ and $\sqrt{b}$, Pythagoras' theorem makes the longest side $\sqrt{a+b}$. Going straight across is shorter than going along the other two sides, so $\sqrt{a+b} < \sqrt{a} + \sqrt{b}$.

A surd is in its simplest form when the number under the square root has no perfect-square factor other than 1. To simplify $\sqrt{n}$, split n into the largest perfect square that divides it, times what is left, and then use rule (b):

$$\sqrt{48} = \sqrt{16 \times 3} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}.$$



The picture shows why. Sixteen small squares, each of area 3 and so of side $\sqrt{3}$, fit together into one square of area 48. Its side is four small sides long, so $\sqrt{48} = 4\sqrt{3}$. Read the other way, $4\sqrt{3} = \sqrt{16} \times \sqrt{3} = \sqrt{48}$. Writing surds as single square roots like this is a quick way to compare their sizes.

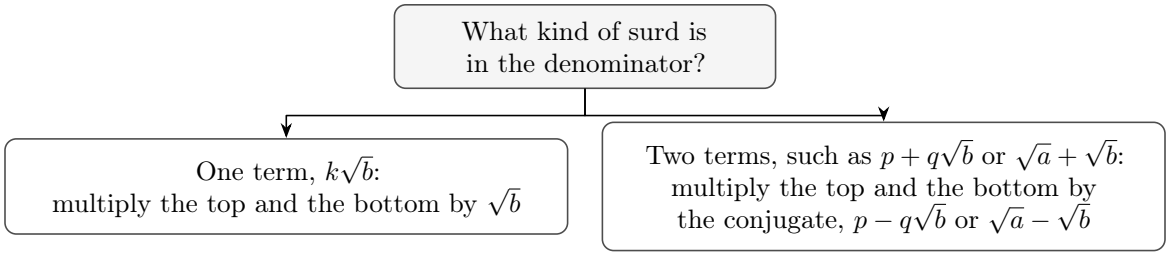
When an expression has several surds, simplify each surd first. Then multiply or divide using rules (b) and (c), and simplify again if the result has a perfect-square factor. Only like surds, such as $5\sqrt{2}$ and $3\sqrt{2}$, can be added or subtracted: $\sqrt{2} + \sqrt{3}$ cannot be simplified.

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Multiply out brackets with surds as you would in algebra, using $\sqrt{a} \times \sqrt{a} = a$. When the two brackets are conjugates, use $(a+b)(a-b) = a^2 - b^2$: no surd is left.

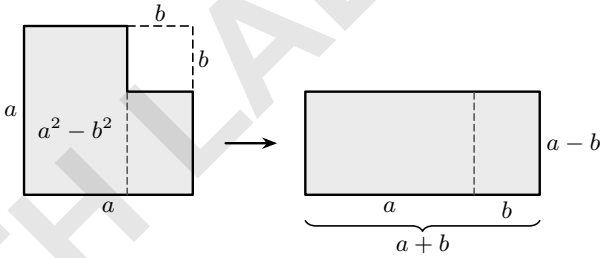
Rationalising the denominator means rewriting a fraction so that its denominator has no surd in it. Do this whenever a surd appears in the denominator of a fraction.



After rationalising, simplify any surd that is left and cancel any common factor.

Why the conjugate works. The shaded L-shape is a square of side a with a square of side b cut from one corner, so its area is $a^2 - b^2$. Cut it into two strips, turn the tall strip on its side and lay the two end to end. They form a rectangle $a + b$ long and $a - b$ wide, so

$$(a+b)(a-b) = a^2 - b^2.$$



When one of the terms is a surd, squaring it removes the root:

$$\left(p + q\sqrt{b}\right)\left(p - q\sqrt{b}\right) = p^2 - q^2 b,$$

and there is no surd left. For example, $(3\sqrt{5} + 4)(3\sqrt{5} - 4) = 45 - 16 = 29$.

To solve an equation with the unknown under a square root: (1) get the square root on its own on one side; (2) square both sides and solve the new equation; (3) substitute *every* answer into the original equation, and reject any answer that does not fit. If there are two square roots, square both sides, get the square root that is left on its own, and square again.

When you square a sum of two square roots, keep the middle term: $\left(\sqrt{A} + \sqrt{B}\right)^2 = A + B + 2\sqrt{AB}$.

Why check? Squaring can bring in an extra answer. Squaring $\sqrt{A} = B$ gives $A = B^2$, but squaring $-\sqrt{A} = B$ gives the same equation. So an answer of the squared equation that makes B negative does not fit the original equation, and it is rejected.

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Comparing terms

Use this method to find unknowns that are rational numbers, such as integers.

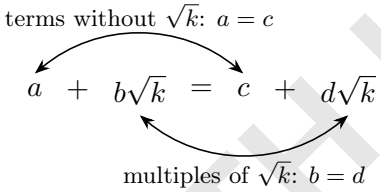
Step 1: Expand and simplify each side. If a surd is in a denominator, rationalise it first. If you are told that a number such as $p + q\sqrt{k}$ is a solution of an equation, substitute it into the equation.

Step 2: Write each side as a number without a surd plus a multiple of $\sqrt{k}$, by grouping the terms without the surd and the terms with it. If one side has no surd term, write it as $0\sqrt{k}$ before matching: for example, $5 = 5 + 0\sqrt{2}$.

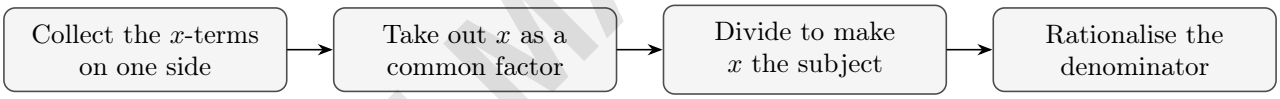
Step 3: Match the two sides: the parts without $\sqrt{k}$ are equal, and so are the multiples of $\sqrt{k}$. This gives two equations.

Step 4: Solve the two equations, simultaneously if both unknowns appear in them. Reject any answer that does not fit the conditions in the question, such as "a and b are positive integers".

Matching is allowed only because the unknowns, and so the parts being matched, are rational (rule (g)).



To solve an equation in which x has surds as its coefficients, make x the subject:



First simplify any surds and expand any brackets, so that like terms can be collected. Every term without x goes to the other side. If there are two unknowns, as in a pair of simultaneous equations, first eliminate one of them, then follow the same four steps.

Word problems with surds

Step 1: Write down the formula that links the quantities in the question, such as area of a triangle = $\frac{1}{2}$ × base × height, volume of a cuboid = base area × height, or Pythagoras' theorem.

Step 2: Substitute the given surd expressions, and expand any squares or products of brackets.

Step 3: Make the unknown quantity the subject. If it is a fraction with a surd in the denominator, rationalise the denominator.

Step 4: Give the answer in the form that the question asks for, with its units.

Before rationalising, divide the top and the bottom of the fraction by any common factor. This keeps the numbers small.

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Exercises

Do not use a calculator.

Section 1: Simplifying Surds

1. Simplify, without using a calculator.

(a) $\sqrt{14} \times \sqrt{21}$ (b) $\dfrac{\sqrt{45} \times \sqrt{40}}{\sqrt{50}}$

2. (a) Write each of $3\sqrt{7}$, $5\sqrt{3}$, $2\sqrt{15}$ and $4\sqrt{5}$ as the square root of a whole number.
(b) Hence, without using a calculator, arrange these four numbers in increasing order.

3. Simplify, without using a calculator.

(a) $\dfrac{\sqrt{72} - \sqrt{18}}{\sqrt{50}}$ (b) $\sqrt{3}\left(\sqrt{12} + 2\sqrt{6}\right) - \sqrt{50}$

4. Expand and simplify.

(a) $\left(2\sqrt{3} + 3\sqrt{2}\right)^2$ (b) $\left(2\sqrt{3} + \sqrt{2}\right)\left(\sqrt{3} - 2\sqrt{2}\right)$

Section 2: Rationalising the Denominator

5. Rationalise the denominator and simplify.

(a) $\dfrac{4}{5\sqrt{18}}$ (b) $\dfrac{10}{2\sqrt{2} + \sqrt{3}}$

6. Simplify, giving each answer with a rational denominator.

(a) $\dfrac{6}{\sqrt{7} - 2} + \dfrac{3}{\sqrt{7} + 2}$ (b) $\dfrac{2\sqrt{5} + \sqrt{15}}{2\sqrt{5} - \sqrt{15}}$

7. Simplify, giving each answer with a rational denominator.

(a) $\left(\sqrt{6} - \dfrac{2}{\sqrt{6}}\right)^2$ (b) $\sqrt{\dfrac{5}{2}} - \sqrt{\dfrac{2}{5}}$

Section 3: Solving by Squaring Both Sides

8. Solve the following equations.

(a) $\dfrac{x + 4}{\sqrt{9x + 22}} = 1$ (b) $3\sqrt{4x + 9} - 2x - 1 = 0$

9. Solve the following equations.

(a) $\sqrt{x^2 + 51} = x + 3$ (b) $3 + \sqrt{x - 4} = \sqrt{x + 23}$

Section 4: Comparing Terms

10. Given that $x = 2 + \sqrt{5}$ is a solution of the equation $x^2 + px + q = 0$, where p and q are integers, find the values of p and q.

11. Find the integers p and q such that $\left(p + \sqrt{2}\right)\left(5 + q\sqrt{8}\right) = 19 + 11\sqrt{2}$.

12. Given that $\dfrac{a + b\sqrt{2}}{3 + \sqrt{2}} = \dfrac{3 + \sqrt{2}}{1 + \sqrt{2}}$, where a and b are integers, find the values of a and b.

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Section 5: Making x the Subject

13. Solve $x\sqrt{3}+1=3x-\sqrt{3}$, giving x in the form $\dfrac{a+b\sqrt{3}}{3}$, where a and b are integers.

14. Solve $\sqrt{2}\,(x+1)=x+3$, giving x in the form $a+b\sqrt{2}$, where a and b are integers.

15. Solve the simultaneous equations

$$x-y=2, \qquad \sqrt{3}\,x+y=6,$$

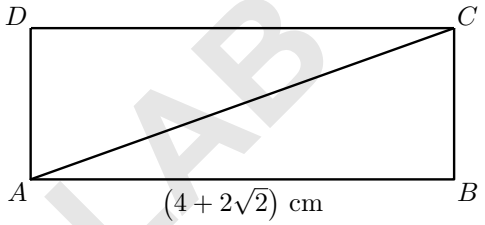
giving x and y in the form $a+b\sqrt{3}$, where a and b are integers.

Section 6: Word Problems

16. The diagram shows a rectangle $ABCD$ in which $AB=\left(4+2\sqrt{2}\right)$ cm. The area of triangle ABC is $\left(4+3\sqrt{2}\right)\text{cm}^2$. Express each of the following in the form $a+b\sqrt{2}$, where a and b are integers.

(a) The length of BC, in cm

(b) AC^2



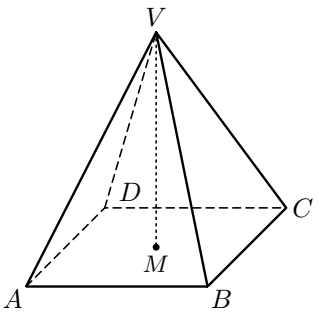
17. A cone has a base radius of $\left(2+\sqrt{2}\right)$ cm and a volume of $\pi\left(14+10\sqrt{2}\right)\text{cm}^3$. Find its height in the form $\left(a+b\sqrt{2}\right)$ cm, where a and b are integers.

18. A hiker walks $\left(10+6\sqrt{2}\right)$ km at a steady speed of $\left(4+\sqrt{2}\right)$ km/h. Without using a calculator, find the time taken, in the form $\left(a+b\sqrt{2}\right)$ hours, where a and b are integers.

19. Do not use a calculator in this question.
The diagram shows a pyramid with a square base $ABCD$.
The vertex V is vertically above M, the centre of the base.
The area of the base is $4\left(2\sqrt{5}-1\right)\text{cm}^2$ and the volume of the pyramid is $\dfrac{4}{3}\left(9+\sqrt{5}\right)\text{cm}^3$. Find

(i) the height VM in the form $\left(a+b\sqrt{5}\right)$ cm, where a and b are integers,

(ii) AM^2 in the form $c+d\sqrt{5}$, where c and d are integers.



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Solutions to Exercises

Section 1 — Simplifying Surds (Questions 1–4)

1(a) $\sqrt{14}\times\sqrt{21}=\sqrt{294}=\sqrt{49\times 6}=\boxed{7\sqrt{6}}$

1(b) $\dfrac{\sqrt{45}\times\sqrt{40}}{\sqrt{50}}=\sqrt{\dfrac{45\times 40}{50}}=\sqrt{36}=\boxed{6}$

2(a) $3\sqrt{7}=\sqrt{9}\times\sqrt{7}=\boxed{\sqrt{63}}$,
$5\sqrt{3}=\sqrt{25}\times\sqrt{3}=\boxed{\sqrt{75}}$,
$2\sqrt{15}=\sqrt{4}\times\sqrt{15}=\boxed{\sqrt{60}}$,
$4\sqrt{5}=\sqrt{16}\times\sqrt{5}=\boxed{\sqrt{80}}$

2(b) $\sqrt{60}<\sqrt{63}<\sqrt{75}<\sqrt{80}$, so
$\boxed{2\sqrt{15},\ 3\sqrt{7},\ 5\sqrt{3},\ 4\sqrt{5}}$

3(a) $\dfrac{\sqrt{72}-\sqrt{18}}{\sqrt{50}}=\dfrac{6\sqrt{2}-3\sqrt{2}}{5\sqrt{2}}=\dfrac{3\sqrt{2}}{5\sqrt{2}}=\boxed{\dfrac{3}{5}}$

3(b) $\sqrt{3}\big(\sqrt{12}+2\sqrt{6}\big)-\sqrt{50}$
$=\sqrt{36}+2\sqrt{18}-5\sqrt{2}$
$=6+6\sqrt{2}-5\sqrt{2}=\boxed{6+\sqrt{2}}$

4(a) $\big(2\sqrt{3}+3\sqrt{2}\big)^2=12+2\big(2\sqrt{3}\big)\big(3\sqrt{2}\big)+18$
$=\boxed{30+12\sqrt{6}}$

4(b) $\big(2\sqrt{3}+\sqrt{2}\big)\big(\sqrt{3}-2\sqrt{2}\big)=6-4\sqrt{6}+\sqrt{6}-4$
$=\boxed{2-3\sqrt{6}}$

Section 2 — Rationalising the Denominator (Questions 5–7)

5(a) $\dfrac{4}{5\sqrt{18}}\times\dfrac{\sqrt{18}}{\sqrt{18}}=\dfrac{4\sqrt{18}}{90}=\dfrac{4\big(3\sqrt{2}\big)}{90}=\boxed{\dfrac{2\sqrt{2}}{15}}$

5(b) $\dfrac{10}{2\sqrt{2}+\sqrt{3}}\times\dfrac{2\sqrt{2}-\sqrt{3}}{2\sqrt{2}-\sqrt{3}}=\dfrac{10\big(2\sqrt{2}-\sqrt{3}\big)}{8-3}$
$=\boxed{4\sqrt{2}-2\sqrt{3}}$

6(a) Over the common denominator
$\big(\sqrt{7}-2\big)\big(\sqrt{7}+2\big)=7-4=3$:
$\dfrac{6}{\sqrt{7}-2}+\dfrac{3}{\sqrt{7}+2}=\dfrac{6\big(\sqrt{7}+2\big)+3\big(\sqrt{7}-2\big)}{3}$
$=\dfrac{6\sqrt{7}+12+3\sqrt{7}-6}{3}=\dfrac{9\sqrt{7}+6}{3}=\boxed{3\sqrt{7}+2}$

6(b) $\dfrac{2\sqrt{5}+\sqrt{15}}{2\sqrt{5}-\sqrt{15}}\times\dfrac{2\sqrt{5}+\sqrt{15}}{2\sqrt{5}+\sqrt{15}}$
$=\dfrac{20+4\sqrt{75}+15}{20-15}=\dfrac{35+20\sqrt{3}}{5}$
$=\boxed{7+4\sqrt{3}}$

7(a) $\left(\sqrt{6}-\dfrac{2}{\sqrt{6}}\right)^2=\big(\sqrt{6}\big)^2-2\big(\sqrt{6}\big)\Big(\dfrac{2}{\sqrt{6}}\Big)+\dfrac{4}{6}$
$=6-4+\dfrac{2}{3}=\boxed{\dfrac{8}{3}}$

7(b) $\sqrt{\dfrac{5}{2}}-\sqrt{\dfrac{2}{5}}=\dfrac{\sqrt{5}}{\sqrt{2}}-\dfrac{\sqrt{2}}{\sqrt{5}}=\dfrac{\sqrt{10}}{2}-\dfrac{\sqrt{10}}{5}$
$=\dfrac{5\sqrt{10}-2\sqrt{10}}{10}=\boxed{\dfrac{3\sqrt{10}}{10}}$

Section 3 — Squaring Both Sides (Questions 8–9)

8(a)
$$x+4=\sqrt{9x+22}$$
$$(x+4)^2=9x+22$$
$$x^2+8x+16=9x+22$$
$$x^2-x-6=0$$
$$(x+2)(x-3)=0$$
Check: $x=-2$: $\dfrac{2}{\sqrt{4}}=1$ ✓; $x=3$: $\dfrac{7}{\sqrt{49}}=1$ ✓.
$\boxed{x=-2 \text{ or } x=3}$

8(b)
$$3\sqrt{4x+9}=2x+1$$
$$9(4x+9)=(2x+1)^2$$
$$36x+81=4x^2+4x+1$$
$$4x^2-32x-80=0$$
$$x^2-8x-20=0$$
$$(x-10)(x+2)=0$$
$$x=10 \quad \text{or} \quad x=-2 \text{ (rej)}$$
Check: $x=10$: $3\sqrt{49}=21=2(10)+1$ ✓; $x=-2$ makes $2x+1<0$.
$\boxed{x=10}$

9(a)
$$\sqrt{x^2+51}=x+3$$
$$x^2+51=x^2+6x+9$$
$$6x=42$$
$$x=7$$
Check: $x=7$: $\sqrt{100}=10=7+3$ ✓.
$\boxed{x=7}$

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9(b) $\big(3+\sqrt{x-4}\big)^2 = x+23$

$$9+6\sqrt{x-4}+x-4 = x+23$$
$$6\sqrt{x-4} = 18$$
$$\sqrt{x-4} = 3$$
$$x-4 = 9$$
$$x = 13$$

Check: $x=13$: $3+\sqrt{9}=6=\sqrt{36}$ ✓.

$\boxed{x=13}$

Section 4 — Comparing Terms (Questions 10–12)

10 Substitute $x=2+\sqrt{5}$, using $\big(2+\sqrt{5}\big)^2 = 9+4\sqrt{5}$:

$9+4\sqrt{5}+p\big(2+\sqrt{5}\big)+q=0$

$(9+2p+q)+(4+p)\sqrt{5}=0+0\sqrt{5}$

Terms with $\sqrt{5}$: $4+p=0$, so $p=-4$.

Terms without $\sqrt{5}$: $9-8+q=0$, so $q=-1$.

$\boxed{p=-4,\ q=-1}$

11 $\sqrt{8}=2\sqrt{2}$, so

$\big(p+\sqrt{2}\big)\big(5+2q\sqrt{2}\big)=19+11\sqrt{2}$

$5p+2pq\sqrt{2}+5\sqrt{2}+4q=19+11\sqrt{2}$

Terms without $\sqrt{2}$: $5p+4q=19$ — (1)

Terms with $\sqrt{2}$: $2pq+5=11$, so $pq=3$ — (2)

From (2), $q=\dfrac{3}{p}$. Substitute into (1):

$$5p+\frac{12}{p}=19$$

$5p^2-19p+12=0$

$(p-3)(5p-4)=0$

$p=3$ or $p=\frac{4}{5}$ (rej) (not an integer)

$\boxed{p=3,\ q=1}$

12 $a+b\sqrt{2} = \dfrac{\big(3+\sqrt{2}\big)^2}{1+\sqrt{2}} = \dfrac{11+6\sqrt{2}}{1+\sqrt{2}}\times\dfrac{1-\sqrt{2}}{1-\sqrt{2}}$

$$= \frac{11-11\sqrt{2}+6\sqrt{2}-12}{1-2}$$
$$= \frac{-1-5\sqrt{2}}{-1} = 1+5\sqrt{2}$$

Comparing terms: $\boxed{a=1,\ b=5}$

Section 5 — Making x the Subject (Questions 13–15)

13 $x\sqrt{3}-3x=-\sqrt{3}-1$

$x\big(\sqrt{3}-3\big)=-\big(\sqrt{3}+1\big)$

$$x = \frac{-\big(\sqrt{3}+1\big)}{\sqrt{3}-3} = \frac{\sqrt{3}+1}{3-\sqrt{3}}$$
$$= \frac{\sqrt{3}+1}{3-\sqrt{3}}\times\frac{3+\sqrt{3}}{3+\sqrt{3}}$$
$$= \frac{3\sqrt{3}+3+3+\sqrt{3}}{9-3} = \frac{6+4\sqrt{3}}{6}$$
$$= \frac{3+2\sqrt{3}}{3}$$

$\boxed{a=3,\ b=2}$

14 $x\sqrt{2}+\sqrt{2}=x+3$

$x\sqrt{2}-x=3-\sqrt{2}$

$x\big(\sqrt{2}-1\big)=3-\sqrt{2}$

$$x = \frac{3-\sqrt{2}}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}$$
$$= \frac{3\sqrt{2}+3-2-\sqrt{2}}{2-1}$$

$\boxed{x=1+2\sqrt{2}}$

15 $x-y=2$ — (1), $\quad\sqrt{3}\,x+y=6$ — (2)

(1) + (2) eliminates y:

$x+\sqrt{3}\,x=8$

$x\big(1+\sqrt{3}\big)=8$

$$x = \frac{8}{1+\sqrt{3}}\times\frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{8\big(\sqrt{3}-1\big)}{3-1}$$
$$= 4\sqrt{3}-4$$

From (1): $y=x-2=4\sqrt{3}-6$

$\boxed{x=-4+4\sqrt{3},\ y=-6+4\sqrt{3}}$

Section 6 — Word Problems (Questions 16–19)

16(a) Area of $\triangle ABC = \frac{1}{2}\times AB\times BC$

$4+3\sqrt{2}=\frac{1}{2}\big(4+2\sqrt{2}\big)BC=\big(2+\sqrt{2}\big)BC$

$$BC = \frac{4+3\sqrt{2}}{2+\sqrt{2}}\times\frac{2-\sqrt{2}}{2-\sqrt{2}}$$
$$= \frac{8-4\sqrt{2}+6\sqrt{2}-6}{4-2} = \frac{2+2\sqrt{2}}{2}$$

$\boxed{BC=1+\sqrt{2}}$

16(b) By Pythagoras' theorem in $\triangle ABC$,

$AC^2 = AB^2+BC^2 = \big(4+2\sqrt{2}\big)^2+\big(1+\sqrt{2}\big)^2$

$= \big(24+16\sqrt{2}\big)+\big(3+2\sqrt{2}\big) = \boxed{27+18\sqrt{2}}$

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17 Volume $= \frac{1}{3}\pi r^2 h$

$$\pi\big(14+10\sqrt{2}\big) = \tfrac{1}{3}\pi\big(2+\sqrt{2}\big)^2 h$$

$$42+30\sqrt{2} = \big(6+4\sqrt{2}\big)h$$

$$h = \frac{21+15\sqrt{2}}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$$

$$= \frac{63-42\sqrt{2}+45\sqrt{2}-60}{9-8}$$

$\boxed{h = \big(3+3\sqrt{2}\big)\text{ cm}}$

18 Time $= \dfrac{\text{distance}}{\text{speed}} = \dfrac{10+6\sqrt{2}}{4+\sqrt{2}} \times \dfrac{4-\sqrt{2}}{4-\sqrt{2}}$

$$= \frac{40-10\sqrt{2}+24\sqrt{2}-12}{16-2} = \frac{28+14\sqrt{2}}{14}$$

$\boxed{\big(2+\sqrt{2}\big)\text{ hours}}$

19(i) Volume $= \frac{1}{3} \times$ base area $\times\, VM$

$$\tfrac{4}{3}\big(9+\sqrt{5}\big) = \tfrac{1}{3} \times 4\big(2\sqrt{5}-1\big) \times VM$$

$$9+\sqrt{5} = \big(2\sqrt{5}-1\big)VM$$

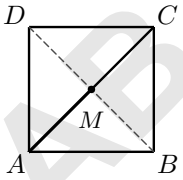
$$VM = \frac{9+\sqrt{5}}{2\sqrt{5}-1} \times \frac{2\sqrt{5}+1}{2\sqrt{5}+1}$$

$$= \frac{18\sqrt{5}+9+10+\sqrt{5}}{20-1}$$

$$= \frac{19+19\sqrt{5}}{19}$$

$\boxed{VM = \big(1+\sqrt{5}\big)\text{ cm}}$

19(ii) M is the midpoint of the diagonal AC of the base.



If the side of the base is s, then $s^2 = 4\big(2\sqrt{5}-1\big)$ and $AC^2 = s^2 + s^2 = 8\big(2\sqrt{5}-1\big)$.

$AM^2 = \big(\frac{1}{2}AC\big)^2 = \frac{1}{4}AC^2 = 2\big(2\sqrt{5}-1\big) = \boxed{-2+4\sqrt{5}}$

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