Linear Inequalities

Solving Simple Inequalities and Using Them in Real-World Problems

Secondary 2 — G3 Mathematics

Targets

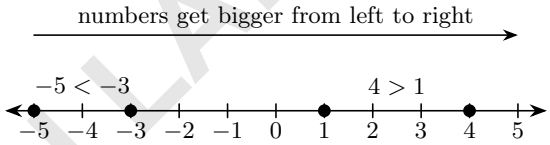
- Solve simple linear inequalities in one unknown, and show the solutions on a number line.
- Use linear inequalities to solve word problems from everyday life.

Notes

1. An **inequality** looks like an equation, but in place of the equals sign it has an **inequality sign**. $3x > 5$, $2x \leq -8$ and $\frac{1}{3}x \geq 4$ are all inequalities.

2. There are four **inequality signs**.

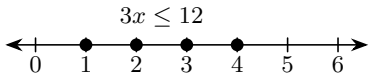
Sign	What it means
$<$	is less than
$>$	is greater than
$\leq$	is less than or equal to
$\geq$	is greater than or equal to



Of two numbers, the one further to the right is the greater.

3. E.g. 1 A minibus is carrying x passengers.
If it carries exactly 8 passengers, then $x = 8$. This is an equation, and it has only one solution for x.
If it carries fewer than 8 passengers, we write $x < 8$. This is an inequality, and it has more than one solution: x can be $0, 1, 2, 3, 4, 5, 6$ or 7. Here x cannot be negative or a fraction, because it counts people.

E.g. 2 Ravi has \$12. He wants to buy x notebooks at \$3 each.
The cost cannot be more than \$12, so we write $3x \leq 12$.
The solutions are $x = 1, 2, 3$ or 4.
$(3\times1 = 3 < 12,\ 3\times2 = 6 < 12,\ 3\times3 = 9 < 12,\ 3\times4 = 12)$
$\therefore$ He can buy $1, 2, 3$ or 4 notebooks. [He wants to buy at least one notebook, so 0 is left out.]



The dots: the numbers of notebooks he can buy.

4. All the values of x that make an inequality true are called the **solutions** of the inequality.

5. If a, b and c are real numbers, then

$a > b$ and $a + c > b + c$ are equivalent.
$a > b$ and $a - c > b - c$ are equivalent.

Here *equivalent* means that whenever one of them is true, so is the other.

E.g. $6 > 2$ so $6 + \mathbf{3} > 2 + \mathbf{3}$ i.e. $9 > 5$
$6 > 2$ so $6 - \mathbf{3} > 2 - \mathbf{3}$ i.e. $3 > -1$

Tip. You may **add** or **subtract** the same number on both sides of an inequality. The number can be positive or negative, and the inequality sign stays as it is.

6. If a, b and c are real numbers and c is **positive**, then

THE MATH LAB

© The Math Lab | For enrolled students only | Do not reproduce or distribute

$a > b$ and $ac > bc$ are equivalent.
$a > b$ and $\dfrac{a}{c} > \dfrac{b}{c}$ are equivalent.

E.g. $6 > 2$ so $6 \times 3 > 2 \times 3$ i.e. $18 > 6$
$6 > 2$ so $\dfrac{6}{2} > \dfrac{2}{2}$ i.e. $3 > 1$

Tip. If you **multiply** or **divide** both sides of an inequality by the same **positive** number, the inequality sign stays as it is.

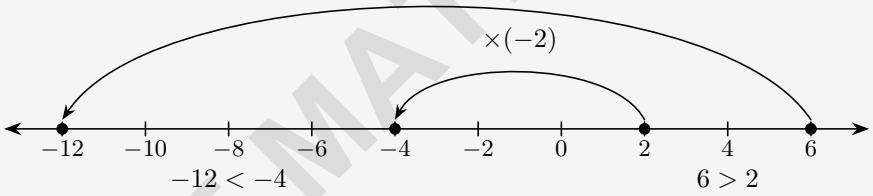
7. If a, b and c is real numbers and c is **negative**, then

$a > b$ and $ac < bc$ are equivalent.
$a > b$ and $\dfrac{a}{c} < \dfrac{b}{c}$ are equivalent.

E.g. $6 > 2$ so $6 \times (-2) < 2 \times (-2)$ i.e. $-12 < -4$
$6 > 2$ so $\dfrac{6}{-2} < \dfrac{2}{-2}$ i.e. $-3 < -1$

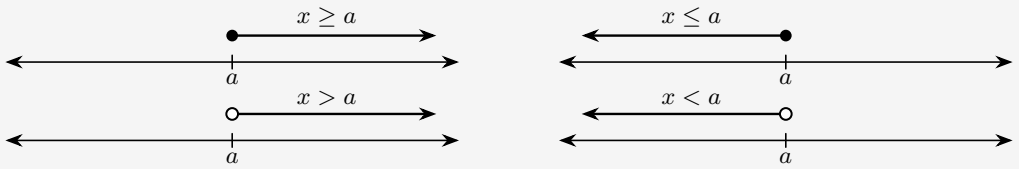
Tip. If you **multiply** or **divide** both sides of an inequality by the same **negative** number, you must **reverse** the inequality sign: $>$ becomes $<$, $\leq$ becomes $\geq$, and so on.

Why does the sign turn round? Multiplying by a negative number moves each number to the other side of 0, so the order of the two numbers is swapped. Below, 6 lies to the right of 2. After multiplying by -2, the answer -12 lies to the left of -4.



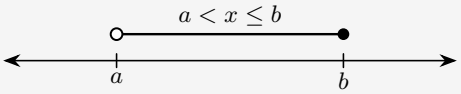
8. We solve a linear inequality in the same way as a linear equation, with one extra rule: whenever we **multiply** or **divide** both sides by a **negative number**, we **reverse** the inequality sign.

Showing Inequalities on a Number Line A **solid** (filled-in) circle means the end value is included ($\geq$ or $\leq$). A **hollow** (empty) circle means it is not included ($>$ or $<$). The arrow points the way the solutions carry on.



A range of values between two numbers, such as $a < x \leq b$, is shown by a line segment from a to b, with the right kind of circle at each end.

To read a range from a number line, look at each end in turn: a solid circle gives $\leq$ or $\geq$, and a hollow circle gives $<$ or $>$.



THE MATH LAB

© The Math Lab | For enrolled students only | Do not reproduce or distribute

Solving a linear inequality, step by step. Not every inequality needs every step.

1. If there are **fractions**, multiply both sides by the LCM of the denominators. The LCM is positive, so the sign stays as it is. Put a bracket round each numerator before you multiply it, e.g. $6 \times \dfrac{x+1}{2} = 3(x+1)$.
2. **Expand** any brackets. Take care with a minus sign in front of a bracket: $-2(x-3) = -2x+6$.
3. Bring the x terms to one side and the numbers to the other side, by adding or subtracting the same thing on both sides. The sign stays as it is.
4. **Simplify** each side, then **divide** both sides by the number in front of x. If that number is **negative**, **reverse** the sign.

If an inequality has both fractions and decimals, write them all as decimals (or all as fractions) first. Dividing both sides by a number is the same as multiplying both sides by its reciprocal: dividing by 4 is the same as multiplying by $\frac{1}{4}$.

Remember. A **prime number** is a whole number greater than 1 that has exactly two different factors, 1 and itself. The first few prime numbers are $2, 3, 5, 7, 11, 13, 17, \ldots$

A **rational number** is a number that can be written as $\dfrac{a}{b}$, where a and b are integers and $b \neq 0$.

For example, $-\frac{3}{4}$, $\frac{2}{5}$ and $2\frac{1}{3}$ are rational numbers.

Every integer is a rational number, e.g. $5 = \frac{5}{1}$ and $-4 = \frac{-4}{1}$.

Every terminating decimal is a rational number, e.g. $0.7 = \frac{7}{10}$ and $2.25 = \frac{9}{4}$.

The **natural numbers** are $1, 2, 3, \ldots$, and the **whole numbers** are $0, 1, 2, 3, \ldots$
The **perfect squares** are $1, 4, 9, 16, 25, \ldots$, and the **perfect cubes** are $1, 8, 27, 64, 125, \ldots$

Finding a particular value from a solution. A question may ask for the greatest or smallest integer, prime number, perfect square, multiple of a number, and so on, that satisfies an inequality.

1. Solve the inequality.
2. Look at the end value. It is included for $\leq$ or $\geq$ (solid circle), and not included for $<$ or $>$ (hollow circle).
3. For the **greatest** value, count down from the end value (move left on the number line); for the **smallest** value, count up from it (move right). The first number of the kind asked for is the answer. Skip the end value itself if it is not included.

For a **rational** number, the end value itself is the answer when it is included, even if it is a fraction. When a question asks for **all** the values of a kind, such as all the prime numbers or all the factors of a number, list every one of them that satisfies the inequality.

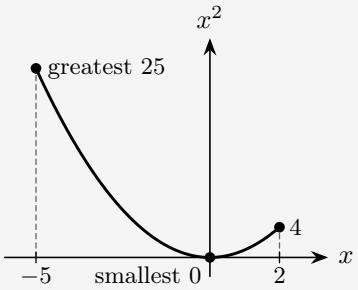
Greatest and smallest values of an expression. When x and y each lie in a range, decide which values of x and y make the expression as large, or as small, as possible.

- $x + y$ is greatest when x and y are both at their greatest, and smallest when both are at their smallest.
- $x - y$ is greatest for the greatest x and the smallest y, and smallest for the smallest x and the greatest y.
- x^2 is never negative. If 0 is in the range of x, the smallest value of x^2 is 0; if not, it is at the end value nearer to 0. The greatest value of x^2 is at the end value further from 0.
- For a product such as xy or a quotient such as $\dfrac{x}{y}$, think about the signs first: positive $\times$ positive and negative $\times$ negative give a positive answer, and positive $\times$ negative gives a negative answer.

THE MATH LAB

© The Math Lab | For enrolled students only | Do not reproduce or distribute

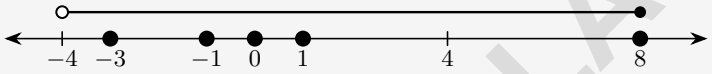
The graph shows x^2 for $-5 \le x \le 2$. The smallest value of x^2 is 0, at $x = 0$, which is not an end of the range. The greatest value is 25, at $x = -5$, the end further from 0; the other end gives only $2^2 = 4$.



A sure method when you are unsure. If you cannot see which values to use, make a short list of values for each variable, keeping only those inside its range:

- the largest and the smallest **positive** values,
- the largest and the smallest **negative** values,
- 0, if 0 is in the range.

For example, if $-4 < x \le 8$, where x is an integer, try $x = -3,\ -1,\ 0,\ 1$ and 8 (the large dots). If x does not have to be an integer, try the two end values of the range and 0.



Then use **trial and error**: work out the expression for each of these values (for every pair of values when there are two variables), and pick out the largest and the smallest answers. It takes a little longer, but it is reliable when you are really unsure.

Careful: we cannot divide by 0, so take care when you try $x = 0$: leave out any value that makes a denominator 0.

Example. For $-4 < x \le 8$, where x is an integer, find the greatest and least values of $\dfrac{24}{x}$.

x	-3	-1	0	1	8
$\dfrac{24}{x}$	-8	-24	not allowed	24	3

Greatest value = 24 (at $x = 1$); least value = -24 (at $x = -1$). The end values -3 and 8 alone would have missed both of them.

One more check: for the square of a bracket that mixes two variables, such as $(x - y)^2$ or $(x + y)^2$, also ask whether the bracket can equal 0 for values inside the ranges. If it can, the smallest value is 0, even when none of the trial values gives 0.

9. A word problem that leads to an inequality is solved with the same steps as a word problem that leads to an equation.

10. Steps for solving a word problem with an inequality:

Step 1
Read the question. Decide what the unknown quantity is and call it a letter, such as x.

→

Step 2
Write an inequality in x that matches the information given.

→

Step 3
Solve the inequality.

→

Step 4
Use the solution to answer the question that was asked.

11. Remember the **inequality signs**, and the everyday words that go with them.

THE MATH LAB

© The Math Lab | For enrolled students only | Do not reproduce or distribute

$<$ means 'is less than'	[also 'fewer than' or 'below']
$>$ means 'is greater than'	[also 'more than', 'above' or 'exceeds']
$\leq$ means 'is less than or equal to'	[also 'at most', 'not more than' or 'no more than']
$\geq$ means 'is greater than or equal to'	[also 'at least' or 'not less than']

E.g. 1 The mass of a parcel, x kg, is more than 20 kg. We write $x > 20$.

E.g. 2 The temperature, x°C, is below 5°C. We write $x < 5$.

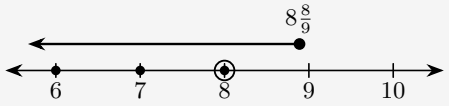
E.g. 3 Mei has at most \$25, so if she has \$$x$, we write $x \leq 25$. [She has \$25 or less.]

E.g. 4 A ride is only for people at least 120 cm tall, so a rider of height x cm has $x \geq 120$. [The height is 120 cm or more.]

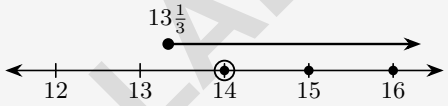
E.g. 5 Four times a number x is not more than 18. We write $4x \leq 18$.

E.g. 6 Half of a number x is greater than 7. We write $\frac{1}{2}x > 7$.

Greatest or least? Read the answer from the number line, do not round off. The answer has to be a whole number that still makes the inequality true. For a **greatest** (maximum) number, take the largest whole number allowed; for a **least** (minimum) number, take the smallest.



$x \leq 8\frac{8}{9}$: greatest whole number 8



$x \geq 13\frac{1}{3}$: least whole number 14

$8\frac{8}{9}$ would round off to 9, but 9 is greater than $8\frac{8}{9}$, so it does not satisfy $x \leq 8\frac{8}{9}$. $13\frac{1}{3}$ would round off to 13, but 13 is less than $13\frac{1}{3}$, so it does not satisfy $x \geq 13\frac{1}{3}$.

More tips for word problems.

- Use the **same units** on both sides of the inequality. For example, if a price is in cents and the money is in dollars, write the dollars as cents first.
- A question that asks for the **possible numbers** wants a list of all the whole numbers that satisfy the inequality. If it says **some**, or at least one, leave 0 out.
- An **average** is the total divided by the number of items. Multiply both sides by the number of items to clear the fraction.
- When a fixed number of items, say n, is made up of two kinds, let x be the number of one kind. Then the number of the other kind is $n - x$.

THE MATH LAB

© The Math Lab | For enrolled students only | Do not reproduce or distribute

Exercises

Section 1: Solving Simple Inequalities

1. Fill in each box with '>', '<' or '=' to make the statement true.

(a) $-5 \square (-2)^3$ (b) $\sqrt{10} \square 3$

2. Solve each inequality, and show its solution on a number line.

(a) $4x > 24$ (b) $-2x \geq 10$ (c) $-3x > 5$

3. Solve the following inequalities.

(a) $-\frac{1}{6}x < -2$ (b) $\frac{7}{8}x \leq -\frac{7}{12}$ (c) $-0.6x < 7.2$

4. Solve the following inequalities.

(a) $\frac{2}{3}x + \frac{2}{5}x \leq -\frac{8}{15}$ (b) $\frac{1}{4}x - 0.7x > 1.8$ (c) $0.4(9x - 3.5x) \geq -55$

5. Solve the following inequalities.

(a) $2 - 9x < -16$ (b) $3(2x + 5) > 7$ (c) $3x - 5 \geq 8x + 15$

6. Solve the following inequalities.

(a) $4(3x - 2) + 2(4x + 4) \leq 30$ (b) $3(4 + 5x) - 6(2 - x) \geq -14$

(c) $\frac{1}{2}(5x - 6) + \frac{1}{4}(x + 12) > 22$

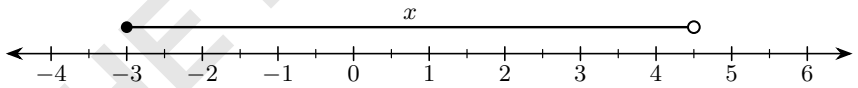
7. Find the greatest integer value of x that satisfies each of the following inequalities.

(a) $2x < -16$ (b) $\frac{2}{5}x \leq 3\frac{1}{5}$

8. Find the smallest integer value of x that satisfies each of the following inequalities.

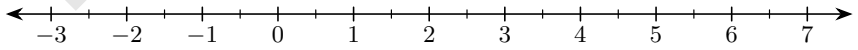
(a) $3x > 13$ (b) $\frac{2}{3}x \geq -14$

9. (a) A range of values of x is shown on the number line below.



Write down the inequalities that describe this range of values of x.

(b) Show $-1\frac{1}{2} < y \leq 6$ on the number line below.



10. Given that $\frac{2}{5}w > 12$, find the smallest value of w for which w is a perfect cube.

11. Given the inequality $7x \leq 60$, find

(a) the greatest rational number x,

(b) the greatest integer value of x,

(c) the greatest prime number x.

12. Given the inequality $\frac{3}{4}x \geq -1.8$, find

(a) the smallest whole number x,

(b) the smallest integer value of x.

THE MATH LAB

© The Math Lab | For enrolled students only | Do not reproduce or distribute

13. (a) Solve the inequality $6x - 5 \leq 21 + 3x$.

(b) Given that x is a natural number, show the solution on a number line.

(c) State the greatest value of x if

(i) x is a perfect square,

(ii) x is a multiple of 3.

14. (a) Solve the inequality $5 - 3x \geq 2(x - 9)$.

(b) Hence, write down the number of positive integers that satisfy $5 - 3x \geq 2(x - 9)$.

15. (a) Solve the inequality $3x > 22 + \frac{5}{3}x$.

(b) Hence, find the smallest possible value of x if

(i) x is a factor of 72,

(ii) x is a common multiple of 4 and 6.

16. (a) Solve the inequality $\dfrac{2-x}{3} \geq \dfrac{3x+1}{4}$.

(b) Write down the greatest integer value of x that satisfies $\dfrac{2-x}{3} \geq \dfrac{3x+1}{4}$.

17. Find the factors of 4290 that satisfy the inequality $4x - \frac{1}{3}x < 51 - \frac{1}{3}$.

18. (a) Express $\dfrac{3x+2}{4} + \dfrac{x-5}{2}$ as a single fraction.

(b) Hence, solve the inequality $\dfrac{3x+2}{4} + \dfrac{x-5}{2} > 8$.

19. Given that g and h are integers such that $-8 \leq g \leq 3$ and $1 \leq h \leq 6$, find

(a) the greatest possible value of $3g + h$,

(b) the greatest possible value of $(g - h)^2$,

(c) the greatest possible value of gh.

20. Given that $-5 \leq x \leq 3$ and $2 \leq y \leq 6$, where x and y are integers, find

(a) the smallest possible value of $(x + y)^2$,

(b) the smallest possible value of xy,

(c) the smallest possible value of $\dfrac{x}{y}$.

21. Given that $-5 \leq x \leq -2$ and $1 \leq y \leq 4$, find

(a) the smallest possible value of $x + y$,

(b) the smallest possible value of $(x - y)^2$,

(c) the greatest possible value of $\dfrac{x}{y}$.

Section 2: Applications of Linear Inequalities in Real-World Contexts

22. A small boat can carry a load of at most 450 kg. On each trip it carries some crates of fish, and each crate has a mass of 40 kg. Find the possible numbers of crates that the boat can carry on one trip.

23. Rachel has at most 2 hours for homework one evening. She first spends $\frac{1}{3}$ hour on an English essay. Then she spends $\frac{1}{8}$ hour on each Mathematics question. Find the greatest number of Mathematics questions she can do that evening.

THE MATH LAB

© The Math Lab | For enrolled students only | Do not reproduce or distribute

24. Sam has a piece of wire 90 cm long. He uses some or all of it to make a square of side x cm and an equilateral triangle of side $(x + 2)$ cm.

(a) Express, in terms of x, the total length of wire that he uses.

(b) Find the greatest possible value of x.

25. A hiker plans to walk not more than 36 km over three days. On the second day she walks twice as far as on the first day. On the third day she walks 6 km less than on the second day. Find the greatest distance she can walk on the first day.

26. A quiz competition has 30 questions. A team scores 4 points for each correct answer and loses 2 points for each wrong answer. No points are given or taken away for a question that is left unanswered. Team Phoenix answered 27 of the questions and scored at least 70 points. Find the least number of questions that the team answered correctly.

27. Consider the sequence $3, 10, 17, 24, 31, \ldots$

(a) Find the general term of the sequence.

(b) Find the 50th term of the sequence.

(c) Find the smallest term of the sequence that is more than 500.

THE MATH LAB

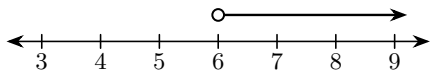
© The Math Lab | For enrolled students only | Do not reproduce or distribute

Solutions to Exercises

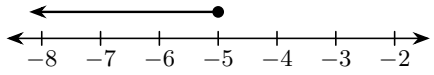
Section 1 — Solving simple inequalities (Questions 1–21)

1 [the number further to the right on the number line is the greater]
(a) $(-2)^3 = -8$, so $-5 \boxed{>} (-2)^3$
(b) $\sqrt{10} \approx 3.16$, so $\sqrt{10} \boxed{>} 3$

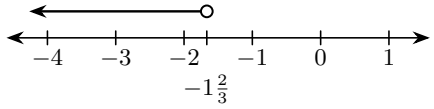
2(a) $4x > 24$; $\boxed{x > 6}$ [divide both sides by 4]



2(b) $-2x \geq 10$; $\dfrac{-2x}{-2} \leq \dfrac{10}{-2}$ [divide both sides by -2 and reverse the inequality sign]; $\boxed{x \leq -5}$



2(c) $-3x > 5$; $\dfrac{-3x}{-3} < \dfrac{5}{-3}$ [divide both sides by -3 and reverse the inequality sign]; $\boxed{x < -1\frac{2}{3}}$



3(a) $-6\left(-\frac{1}{6}x\right) > -6(-2)$ [multiply by -6, reverse sign]; $\boxed{x > 12}$

3(b) $\frac{8}{7}\left(\frac{7}{8}x\right) \leq \frac{8}{7}\left(-\frac{7}{12}\right)$ [multiply by $\frac{8}{7}$]; $x \leq -\frac{8}{12}$; $\boxed{x \leq -\frac{2}{3}}$

3(c) $\dfrac{-0.6x}{-0.6} > \dfrac{7.2}{-0.6}$ [divide by -0.6, reverse sign]; $\boxed{x > -12}$

4(a) $\dfrac{10x + 6x}{15} \leq -\dfrac{8}{15}$ [the LCM of 3 and 5 is 15]; $\frac{16}{15}x \leq -\frac{8}{15}$; $x \leq \frac{15}{16}\left(-\frac{8}{15}\right)$ [multiply by $\frac{15}{16}$]; $\boxed{x \leq -\frac{1}{2}}$

4(b) $0.25x - 0.7x > 1.8$ [write the fraction as a decimal]; $-0.45x > 1.8$; $\dfrac{-0.45x}{-0.45} < \dfrac{1.8}{-0.45}$ [divide by -0.45, reverse sign]; $\boxed{x < -4}$

4(c) $0.4(5.5x) \geq -55$; $2.2x \geq -55$; $x \geq \frac{-55}{2.2}$; $\boxed{x \geq -25}$

5(a) $-9x < -18$ [subtract 2 from both sides]; $\dfrac{-9x}{-9} > \dfrac{-18}{-9}$ [divide by -9, reverse sign]; $\boxed{x > 2}$

5(b) $6x + 15 > 7$; $6x > -8$; $x > -\frac{8}{6}$; $\boxed{x > -1\frac{1}{3}}$

5(c) $3x - 8x \geq 15 + 5$; $-5x \geq 20$; $\dfrac{-5x}{-5} \leq \dfrac{20}{-5}$ [divide by -5, reverse sign]; $\boxed{x \leq -4}$

6(a) $12x - 8 + 8x + 8 \leq 30$; $20x \leq 30$; $x \leq \frac{30}{20}$; $\boxed{x \leq 1\frac{1}{2}}$

6(b) $12 + 15x - 12 + 6x \geq -14$; $21x \geq -14$; $x \geq -\frac{14}{21}$; $\boxed{x \geq -\frac{2}{3}}$

6(c) $2(5x - 6) + (x + 12) > 88$ [multiply both sides by 4, the LCM of 2 and 4]; $10x - 12 + x + 12 > 88$; $11x > 88$; $\boxed{x > 8}$

7(a) $2x < -16$; $x < -8$. Greatest integer $= \boxed{-9}$ [-8 is not included]

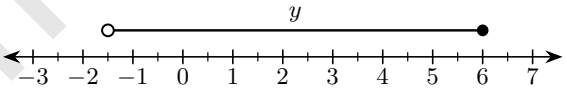
7(b) $\frac{5}{2}\left(\frac{2}{5}x\right) \leq \frac{5}{2}\left(\frac{16}{5}\right)$; $x \leq 8$. Greatest integer $= \boxed{8}$

8(a) $3x > 13$; $x > 4\frac{1}{3}$. Smallest integer $= \boxed{5}$

8(b) $\frac{3}{2}\left(\frac{2}{3}x\right) \geq \frac{3}{2}(-14)$; $x \geq -21$. Smallest integer $= \boxed{-21}$

9(a) $\boxed{-3 \leq x < 4\frac{1}{2}}$ [solid circle at -3: included; hollow circle at $4\frac{1}{2}$: not included]

9(b)



[hollow circle at $-1\frac{1}{2}$: not included; solid circle at 6: included]

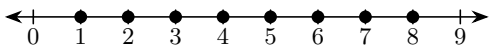
10 $\frac{5}{2}\left(\frac{2}{5}w\right) > \frac{5}{2}(12)$; $w > 30$.
Perfect cubes: $1, 8, 27, 64, \ldots$
Smallest perfect cube $w = \boxed{64}$ [$27 < 30$]

11 $7x \leq 60$; $x \leq 8\frac{4}{7}$.
(a) $\boxed{8\frac{4}{7}}$ (b) $\boxed{8}$ (c) $\boxed{7}$ [8 is not prime]

12 $\frac{4}{3}\left(\frac{3}{4}x\right) \geq \frac{4}{3}(-1.8)$; $x \geq -2.4$.
(a) $\boxed{0}$ [whole numbers are $0, 1, 2, \ldots$] (b) $\boxed{-2}$ [$-3 < -2.4$]

13(a) $6x - 3x \leq 21 + 5$; $3x \leq 26$; $\boxed{x \leq 8\frac{2}{3}}$

13(b) $x = 1, 2, 3, 4, 5, 6, 7, 8$



13(c) (i) $\boxed{4}$ [$9 > 8\frac{2}{3}$] (ii) $\boxed{6}$ [$9 > 8\frac{2}{3}$]

14(a) $5 - 3x \geq 2x - 18$ [expand the bracket]; $-3x - 2x \geq -18 - 5$; $-5x \geq -23$; $\dfrac{-5x}{-5} \leq \dfrac{-23}{-5}$ [divide by -5, reverse sign]; $\boxed{x \leq 4\frac{3}{5}}$

14(b) The positive integers are $1, 2, 3, 4$, so the number of them is $\boxed{4}$

THE MATH LAB

© The Math Lab | For enrolled students only | Do not reproduce or distribute

15(a) $3x - \frac{5}{3}x > 22$; $\frac{4}{3}x > 22$; $x > \frac{3}{4}(22)$; $\boxed{x > 16\frac{1}{2}}$

15(b) (i) Factors of 72: $1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72$. Smallest factor greater than $16\frac{1}{2}$: $\boxed{18}$
(ii) The common multiples of 4 and 6 are the multiples of 12: $12, 24, 36, \ldots$ Smallest: $\boxed{24}$ [$12 < 16\frac{1}{2}$]

16(a) $4(2 - x) \geq 3(3x + 1)$ [multiply both sides by 12]; $8 - 4x \geq 9x + 3$; $-13x \geq -5$; $x \leq \frac{-5}{-13}$ [divide by -13, reverse sign]; $\boxed{x \leq \frac{5}{13}}$

16(b) Greatest integer value of $x = \boxed{0}$ [$1 > \frac{5}{13}$]

17 $\frac{11}{3}x < 50\frac{2}{3}$; $\frac{11}{3}x < \frac{152}{3}$; $x < \frac{3}{11}\left(\frac{152}{3}\right)$; $x < 13\frac{9}{11}$.
$4290 = 2 \times 3 \times 5 \times 11 \times 13$; its factors are $1, 2, 3, 5, 6, 10, 11, 13, 15, \ldots$
Factors that satisfy the inequality:
$\boxed{1,\ 2,\ 3,\ 5,\ 6,\ 10,\ 11,\ 13}$

18(a)
$$\frac{(3x + 2) + 2(x - 5)}{4} = \frac{3x + 2 + 2x - 10}{4} = \boxed{\frac{5x - 8}{4}}$$

18(b) $\frac{5x - 8}{4} > 8$; $5x - 8 > 32$ [multiply by 4]; $5x > 40$; $\boxed{x > 8}$

19 Values to try: $g = -8,\ -1,\ 0,\ 1,\ 3$; $h = 1,\ 6$
(a) $3g + h = 3(3) + 6 = \boxed{15}$ [greatest g and greatest h]
(b) $(g - h)^2 = (-8 - 6)^2 = (-14)^2 = \boxed{196}$ [make $g - h$ as far from 0 as possible: smallest g − greatest h; the other way, $3 - 1 = 2$, is much smaller in size]
(c) $gh = (3)(6) = \boxed{18}$ [positive × positive; with g negative the product is negative]

20 Values to try: $x = -5,\ -1,\ 0,\ 1,\ 3$; $y = 2,\ 6$
(a) $x + y$ can equal 0, e.g. $x = -2,\ y = 2$. So the smallest value of $(x + y)^2$ is $\boxed{0}$ [a square is never negative]
(b) $xy = (-5)(6) = \boxed{-30}$
(c) $\frac{x}{y} = \frac{-5}{2} = \boxed{-2\frac{1}{2}}$ [most negative x over the smallest y]

21 Values to try: $x = -5,\ -2$; $y = 1,\ 4$
(a) $x + y = -5 + 1 = \boxed{-4}$ [smallest x and smallest y]
(b) $x - y$ lies between $-5 - 4 = -9$ and $-2 - 1 = -3$, so it is never 0. It is closest to 0 when $x = -2$ and $y = 1$: $(x - y)^2 = (-3)^2 = \boxed{9}$
(c) $\frac{x}{y} = \frac{-2}{4} = \boxed{-\frac{1}{2}}$ [$\frac{x}{y}$ is always negative, so take x closest to 0 and the greatest y]

Section 2 — Applications (Questions 22–27)

22 Let the number of crates be x.
$40x \leq 450$; $x \leq 11\frac{1}{4}$
$\therefore$ Possible numbers of crates: $\boxed{1, 2, 3, \ldots, 11}$
[it carries some crates, so 0 is left out]

23 Let the number of Mathematics questions be x.
$\frac{1}{3} + \frac{1}{8}x \leq 2$; $\frac{1}{8}x \leq 1\frac{2}{3}$; $x \leq 8\left(\frac{5}{3}\right)$; $x \leq 13\frac{1}{3}$
$\therefore$ Greatest number of questions $= \boxed{13}$

24(a) Total length $= 4x + 3(x + 2) = \boxed{(7x + 6)\text{ cm}}$

24(b) $7x + 6 \leq 90$; $7x \leq 84$; $x \leq 12$
Greatest possible value of $x = \boxed{12}$

25 Let the distance on the first day be x km. Second day $= 2x$ km; third day $= (2x - 6)$ km.
$x + 2x + (2x - 6) \leq 36$; $5x - 6 \leq 36$; $5x \leq 42$; $x \leq 8.4$
$\therefore$ Greatest distance on the first day $= \boxed{8.4\text{ km}}$

26 Let the number of correct answers be x. Wrong answers $= 27 - x$.
$4x - 2(27 - x) \geq 70$; $4x - 54 + 2x \geq 70$; $6x \geq 124$; $x \geq 20\frac{2}{3}$
$\therefore$ Least number of correct answers $= \boxed{21}$
[check: 21 correct and 6 wrong give $84 - 12 = 72$; 20 correct and 7 wrong give only $80 - 14 = 66$]

27(a) The terms go up by 7 each time, so $T_n = 7n + c$.
$T_1 = 7 + c = 3$, so $c = -4$.
$\boxed{T_n = 7n - 4}$

27(b) $T_{50} = 7(50) - 4 = \boxed{346}$

27(c) $7n - 4 > 500$; $7n > 504$; $n > 72$, so $n = 73$.
Smallest term $= 7(73) - 4 = \boxed{507}$ [$T_{72} = 500$ is not more than 500]

THE MATH LAB

© The Math Lab | For enrolled students only | Do not reproduce or distribute