Real Numbers and Order of Operations

Integers, Fractions, Decimals, Rational and Irrational Numbers

Secondary 1 — G3 Mathematics

Notes

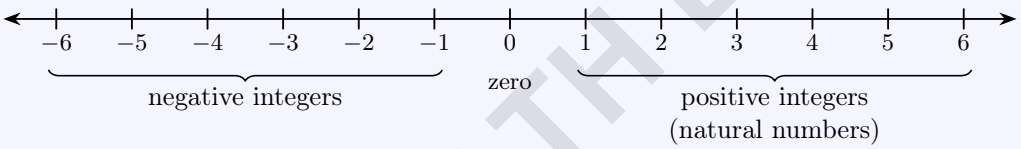
Negative Numbers and Integers

Numbers less than zero, such as -1, -5 and -12, are **negative numbers**. They are written with a minus sign ($-$) in front.

The **integers** are the whole numbers and their negatives: $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$

- **Positive integers** are the integers greater than 0: $1, 2, 3, 4, \ldots$ They are also called **natural numbers**.
- **Negative integers** are the integers less than 0: $-1, -2, -3, \ldots$
- **Zero** is an integer, but it is neither positive nor negative.
- **Whole numbers** are $0, 1, 2, 3, \ldots$, that is, the natural numbers together with 0. A whole number is never negative, and 0 is a whole number but not a natural number.

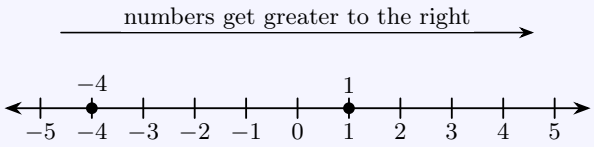
On a **number line**, the integers are marked at equal steps. The arrows at the two ends show that the line carries on without end in both directions.



Comparing Numbers on the Number Line

Of two numbers on the number line, the one further to the **right** is the **greater** one.

For example, 1 lies to the right of -4, so 1 is greater than -4: we write $1 > -4$. In the same way, -4 is less than 1: we write $-4 < 1$.



$>$ means "is greater than"

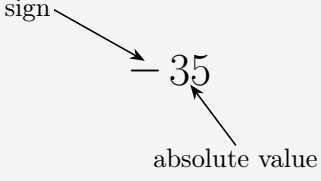
$<$ means "is less than"

$\geqslant$ means "is greater than or equal to"

$\leqslant$ means "is less than or equal to"

The signs $>$, $<$, $\geqslant$ and $\leqslant$ are called **inequality signs**.

Sign and absolute value. Every number other than 0 has a size, called its **absolute value**, and a sign, $+$ or $-$. The number -35 has the sign $-$ and the absolute value 35. We usually leave out the $+$ sign of a positive number: $+18$ is written simply as 18.



THE MATH LAB

© The Math Lab | For enrolled students only | Do not reproduce or distribute

Prime Numbers, Composite Numbers, Perfect Squares and Perfect Cubes

A **prime number** is a natural number with exactly two different factors: 1 and the number itself. The prime numbers are $2, 3, 5, 7, 11, 13, 17, 19, 23, 29, \ldots$ The smallest prime is 2, and it is the only even prime. The numbers 0 and 1, and all negative numbers, are **not** prime.

A **composite number** is a natural number with more than two different factors, such as 4, 6, 8, 9, 10, 12, 14 and 15. Every natural number greater than 1 is either prime or composite. The number 1 is neither.

A **perfect square** is the square of a natural number: $1 = 1^2$, $4 = 2^2$, $9 = 3^2$, 16, 25, ... A **perfect cube** is the cube of a natural number: $1 = 1^3$, $8 = 2^3$, $27 = 3^3$, 64, 125, ... So 1 is both a perfect square and a perfect cube, while 0 is neither.

Showing numbers on a number line. Mark each number of the set with a dot. The signs $\geqslant$ and $\leqslant$, and the words "at least" and "at most", include the end number; $>$ and $<$ do not. If the numbers carry on without end, mark the first few and write "...".

Adding Two Integers

Signs of the numbers	What to do	Examples
Same sign (both + or both −)	**Add** the absolute values and keep the common sign. $(+a)+(+b)=+(a+b)$ $(-a)+(-b)=-(a+b)$	$9+14=23$ $-9-14=-(9+14)=-23$ $(-6)+(-5)=-(6+5)=-11$
Different signs (one +, one −)	**Subtract** the smaller absolute value from the larger one. The answer takes the sign of the number with the larger absolute value.	$-15+6=-(15-6)=-9$ $-6+15=15-6=9$ $11+(-3)=11-3=8$ $2+(-10)=-(10-2)=-8$

In $-15+6$ the answer is negative because the absolute value of -15 is larger than that of 6.

Subtracting an Integer

Subtracting a number is the same as **adding its opposite**:

$$a-b=a+(-b).$$

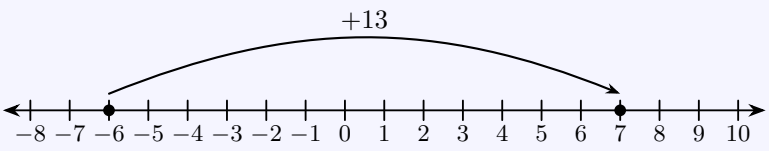
When two signs stand side by side, two like signs give + and two unlike signs give −:

$$+(+b)=+b, \qquad +(-b)=-b, \qquad -(+b)=-b, \qquad -(-b)=+b.$$

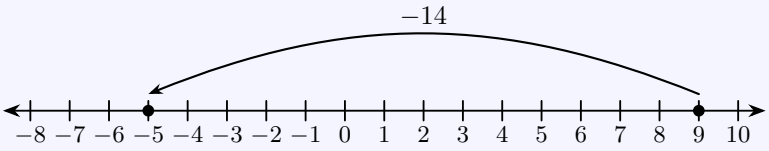
For example, $6-14=6+(-14)=-(14-6)=-8$, $-9-8=-9+(-8)=-(9+8)=-17$, $-5-(-12)=-5+12=7$ and $4-(-11)=4+11=15$.

On a number line, adding a positive number moves you to the right, and subtracting a positive number moves you to the left:

$-6+13=7$



$9-14=-5$



THE MATH LAB

© The Math Lab | For enrolled students only | Do not reproduce or distribute

Finding a missing number. Undo what was done to the box. If $8 + \square = -2$, the box holds $-2 - 8 = -10$. If $\square - 6 = -4$, the box holds $-4 + 6 = 2$. If $1 - \square = 13$, the box holds $1 - 13 = -12$; check: $1 - (-12) = 1 + 12 = 13$.

Multiplying and Dividing Integers

Rule for signs	Multiplication	Division
$(+) \times (+) = (+)$, $(+) \div (+) = (+)$	$5 \times 3 = 15$	$21 \div 7 = 3$
$(-) \times (-) = (+)$, $(-) \div (-) = (+)$	$(-4) \times (-7) = +(4 \times 7) = 28$	$(-32) \div (-4) = +(32 \div 4) = 8$
$(+) \times (-) = (-)$, $(+) \div (-) = (-)$	$6 \times (-5) = -(6 \times 5) = -30$	$45 \div (-9) = -(45 \div 9) = -5$
$(-) \times (+) = (-)$, $(-) \div (+) = (-)$	$(-11) \times 3 = -(11 \times 3) = -33$	$(-42) \div 6 = -(42 \div 6) = -7$

Same signs give a **positive** answer; **different** signs give a **negative** answer. When more than two numbers are multiplied, count the negative signs: an even number of them gives a positive answer, and an odd number gives a negative answer.

Factors, Powers and Roots with Negative Numbers

- An integer has negative factors as well as positive ones. The factors of 10 are ± 1, ± 2, ± 5 and ± 10. The sign $\pm$ means "plus or minus": ± 2 stands for 2 or -2.
- Since $7 \times 7 = 49$ and $(-7) \times (-7) = 49$, the number 49 has **two** square roots: $\sqrt{49} = 7$ and $-\sqrt{49} = -7$, written together as $\pm\sqrt{49} = \pm 7$. In the same way, the square roots of 64 are ± 8. The sign $\sqrt{\ }$ on its own always means the positive square root.
- A number has only **one** cube root. For example, $\sqrt[3]{-125} = -5$, since $(-5) \times (-5) \times (-5) = -125$.
- An even power of a negative number is positive, and an odd power is negative: $(-2)^4 = (-2) \times (-2) \times (-2) \times (-2) = 16$, while $(-2)^5 = -32$. Take care: $-2^4 = -(2 \times 2 \times 2 \times 2) = -16$ is not the same as $(-2)^4 = 16$.

Order of Operations (BODMAS)

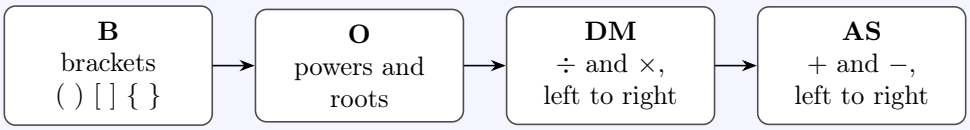
Work out every expression in the order given by BODMAS:

Step 1: (B) Work out what is inside the **brackets** first. When there are brackets inside brackets, such as ( ) inside [ ] inside { }, start with the innermost pair and work outwards.

Step 2: (O) Next work out the **orders**: powers, squares, cubes, square roots and cube roots.

Step 3: (DM) Then do the **divisions and multiplications**, from left to right.

Step 4: (AS) Last of all, do the **additions and subtractions**, from left to right.



A fraction bar acts like a pair of brackets: work out the top and the bottom first, and then divide. Inside each pair of brackets, use the same order: powers and roots first, then × and ÷, then + and −.

THE MATH LAB

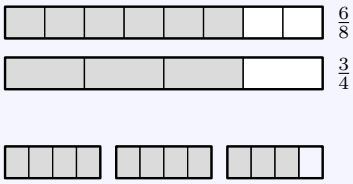
© The Math Lab | For enrolled students only | Do not reproduce or distribute

Recall: Types of Fractions

$\frac{6}{8} = \frac{3}{4}$, so $\frac{6}{8}$ and $\frac{3}{4}$ are **equivalent fractions**. $\frac{3}{4}$ is in its **simplest form** (or **lowest terms**): its numerator and denominator have no common factor except 1.

In a **proper fraction** the numerator is less than the denominator, e.g. $\frac{2}{5}$, $\frac{4}{9}$ and $\frac{7}{12}$. In an **improper fraction** the numerator is greater than or equal to the denominator, e.g. $\frac{11}{4}$. A **mixed number** such as $2\frac{3}{4}$ is a whole number plus a proper fraction:

$$\frac{22}{8} = \frac{11}{4} = 2\frac{3}{4}.$$



2 wholes and 3 quarters = 11 quarters: $2\frac{3}{4} = \frac{11}{4}$

The Four Operations on Fractions

It is usually safest to change each mixed number to an improper fraction before you start. The sign rules for integers work for fractions too.

To **add or subtract** fractions, first rewrite them with the **same denominator**. The easiest one to use is the lowest common multiple (LCM) of the denominators.

Example:
$$\begin{aligned} 5\frac{1}{3} - 2\frac{5}{6} &= \frac{16}{3} - \frac{17}{6} && \text{[change to improper fractions]} \\ &= \frac{32}{6} - \frac{17}{6} && \text{[use the same denominator]} \\ &= \frac{15}{6} = \frac{5}{2} && \text{[subtract the numerators; simplify]} \\ &= 2\frac{1}{2} && \text{[change back to a mixed number]} \end{aligned}$$

You can also add or subtract mixed numbers by dealing with the whole-number parts and the fraction parts separately: $5\frac{1}{3} - 2\frac{5}{6} = (5-2) + \left(\frac{2}{6} - \frac{5}{6}\right) = 3 - \frac{3}{6} = 3 - \frac{1}{2} = 2\frac{1}{2}$.

To **multiply or divide** fractions, the denominators do not need to be the same. **Multiply the numerators together and the denominators together**. To divide by a fraction, change $\div$ to $\times$ and turn the second fraction upside down (swap its numerator and denominator):

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}, \qquad \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c}, \qquad \text{where } a, b, c, d \text{ are integers and } b, c, d \neq 0.$$

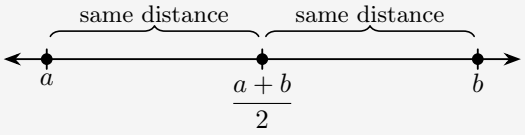
Cancel any common factors of a numerator and a denominator before you multiply, and give the answer in its simplest form.

Example:
$$\begin{aligned} 2\frac{2}{5} \times \frac{5}{9} \times 3 \div 8 &= \frac{12}{5} \times \frac{5}{9} \times \frac{3}{1} \times \frac{1}{8} && \text{[improper fractions; } \div 8 \text{ becomes } \times \tfrac{1}{8}\text{]} \\ &= \frac{\overset{4}{\cancel{12}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{9}}} \times \frac{3}{1} \times \frac{1}{8} && \text{[cancel common factors]} \\ &= \frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{1}{\cancel{3}}}{1} \times \frac{1}{\underset{2}{\cancel{8}}} && \text{[cancel again]} \\ &= \frac{1}{2} \end{aligned}$$

THE MATH LAB

© The Math Lab | For enrolled students only | Do not reproduce or distribute

The number halfway between two numbers. The number exactly in the middle of two numbers is their **average**: add the two numbers and divide by 2. The number $\dfrac{a+b}{2}$ is as far from a as it is from b. So the two numbers add up to twice the number halfway between them.



If one of the numbers is a decimal and the answer is wanted as a fraction, change the decimal to a fraction first, for example $0.75 = \frac{3}{4}$.

The Four Operations on Decimals

The sign rules for integers work for decimals too. Decide the sign of the answer first, and then work with the absolute values of the numbers.

- To **add or subtract** decimals, set them out in columns with the **decimal points in a line**. Fill in zeros so that both numbers have the same number of decimal places.
- To **multiply** decimals, first multiply them as if they were whole numbers. The answer has as many decimal places as the two numbers have altogether.
- To **divide** by a decimal, move the decimal point of both numbers the same number of places to the right until the number you divide by is a whole number. Then divide, keeping the decimal point of the answer directly above the decimal point of the number being divided.

Rational Numbers

A **rational number** is a number that can be written as a fraction with an integer on top and an integer at the bottom:

$$\text{a rational number can be written as } \frac{a}{b}, \text{ where } a \text{ and } b \text{ are integers and } b \neq 0.$$

For example, $-\dfrac{4}{9}$, $\dfrac{3}{8}$, $\dfrac{7}{2}$ and $2\dfrac{1}{3} = \dfrac{7}{3}$ are rational numbers.

- Every integer is rational, because an integer n can be written as $\dfrac{n}{1}$. For example, $5 = \dfrac{5}{1}$ and $-8 = \dfrac{-8}{1}$.
- Every terminating decimal is rational: $0.7 = \dfrac{7}{10}$ and $2.45 = \dfrac{245}{100}$; so is -1.6.
- Every recurring decimal, such as $0.\dot{4}$ or $2.\dot{1}\dot{5}$, is rational too. The number $\dfrac{22}{7}$ is rational, since it is one integer divided by another.

Writing a Fraction as a Decimal

To write a fraction as a decimal, divide the numerator by the denominator.

- If the division comes to an end, the decimal has a fixed number of digits. It is a **terminating decimal**, e.g. $\dfrac{3}{16} = 0.1875$.
- If a digit, or a group of digits, repeats for ever, the decimal is a **recurring** (or **repeating**) **decimal**. A dot is placed over the repeating digit. When a group of two or more digits repeats, the dots go over the first and the last digits of the group:

$$\frac{2}{3} = 0.666\ldots = 0.\dot{6}, \qquad \frac{4}{37} = 0.108\,108\,108\ldots = 0.\dot{1}0\dot{8},$$

THE MATH LAB

© The Math Lab | For enrolled students only | Do not reproduce or distribute

$$\frac{3}{7} = 0.428\,571\,428\,571\ldots = 0.\dot{4}2857\dot{1}.$$

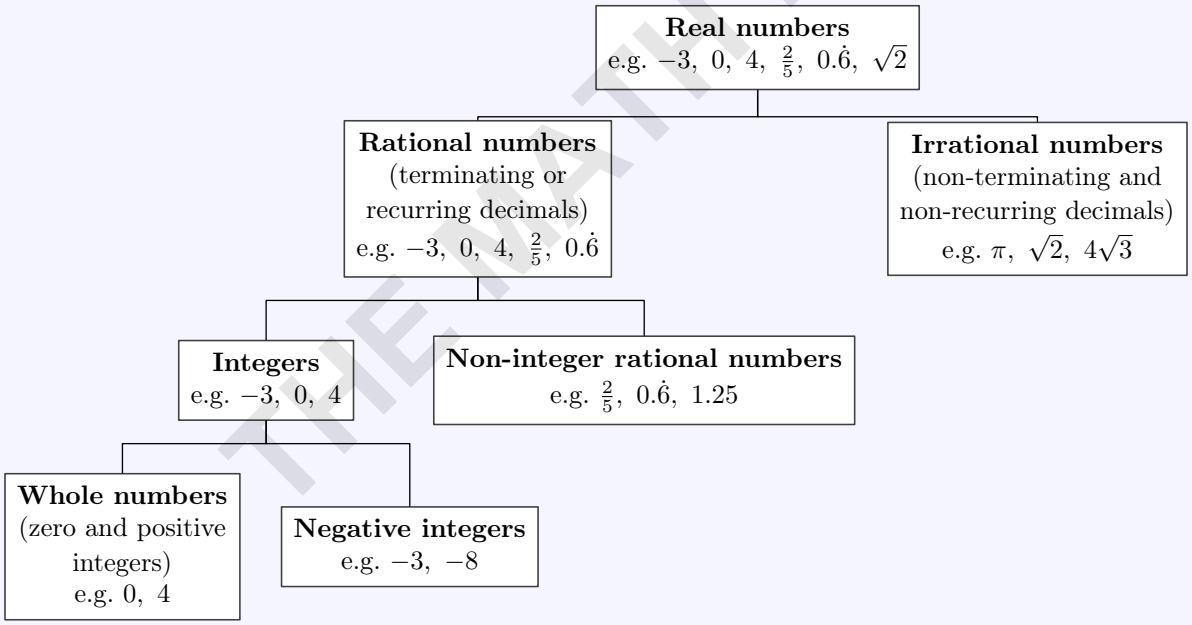
- In the division, as soon as a remainder comes back that you have had before, the digits of the answer repeat from that point.
- For a mixed number, write the fraction part as a decimal and put the whole number in front.

Irrational Numbers and Real Numbers

Some decimals go on for ever **without** any repeating pattern. They are **non-terminating, non-recurring decimals**. One example is π, the circumference of a circle divided by its diameter: $\pi = 3.141\,592\,65\ldots$ Another is $\sqrt{2} = 1.414\,213\,56\ldots$ Numbers that cannot be written as $\frac{a}{b}$, where a and b are integers and $b \neq 0$, are called **irrational numbers**. For example, π, $\sqrt{2}$, $\sqrt{7}$, $\sqrt{10}$, $-\sqrt[3]{9}$ and $4\sqrt{3}$ are irrational. The square root of a whole number that is not a perfect square, such as $\sqrt{2}$, $\sqrt{3}$ or $\sqrt{5}$, is irrational, and so is π multiplied or divided by any rational number other than 0. Check fractions and decimals before you decide: $\sqrt{0.25} = 0.5$ and $\sqrt{\frac{4}{9}} = \frac{2}{3}$ are rational.

π is not rational: it is not one integer divided by another. The rational numbers $\frac{22}{7}$, 3.14 and 3.142 are only **approximations** to π.

The **real numbers** are all the numbers on the number line. They split into rational and irrational numbers as shown below.



Before you sort numbers into types, simplify every number that can be simplified, and then decide using the simplified value. For example, $\sqrt{100} = 10$, so $\sqrt{100}$ is a natural number, a whole number, an integer and a rational number.

Comparing and ordering numbers: tips and reminders.

- **Ascending** order runs from the smallest number to the largest; **descending** order runs from the largest to the smallest.
- To compare numbers easily, write each of them as a decimal, rounded to 5 or 6 decimal places. You may use a calculator to do this.
- In your final answer, write each number in its **original form**, not as the rounded decimal.

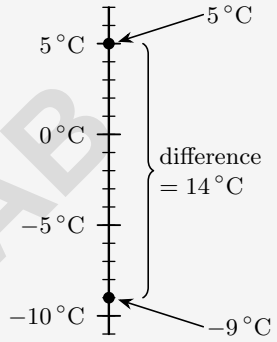
© The Math Lab | For enrolled students only | Do not reproduce or distribute

- Of two negative numbers, the one with the larger absolute value is the smaller number: -6 is less than -5.
- Marking the numbers on a number line also shows their order: numbers get greater from left to right.
- To find the two whole numbers that a square root lies between, look for the perfect squares on either side. For example, $25 < 30 < 36$, so $5 < \sqrt{30} < 6$. Cube roots work in the same way with perfect cubes: $8 < 20 < 27$, so $2 < \sqrt[3]{20} < 3$.

Using a calculator. Key in the whole expression in one go, using brackets and the fraction key so that the calculator follows the correct order of operations. Write down the full calculator display first, and then round it to the accuracy asked for. Keep any zero at the end of a rounded answer: 4.6038... correct to 2 decimal places is 4.60.

Negative numbers in everyday life. Negative numbers describe amounts below a zero level: temperatures below 0 °C, heights below sea level, floors below the ground floor, money owed, marks taken away.

The **difference** between two temperatures is the higher temperature minus the lower one. On a thermometer, it is the distance between the two readings.



THE MATH LAB

Solving Real-World Problems

- **A rise or a fall.** Add a rise and subtract a fall. The highest temperature is the lowest plus the difference between them; the lowest is the highest minus the difference.
- **Above and below sea level.** Heights above sea level are positive and depths below it are negative. The vertical distance between two points is the higher height minus the lower one.
- **Gains and losses.** Write gains (profits, marks given) as positive numbers and losses (money lost, marks taken away) as negative numbers, and add them. A negative total is an overall loss.
- **Average.** The average of several values is their sum, signs included, divided by the number of values. The value midway between two values is their average.
- **A steady rate of change.** Find the change for one unit (one hour, one minute or one metre), and multiply it by the number of units. To find how many units a change takes, divide the change by the change for one unit.
- **A fraction of the remainder.** Turn a fraction "of the remainder" into a fraction of the whole by multiplying it by the fraction that remained. Work with fractions of the whole, use the amount you are given to find the value of one part, and then multiply.

© The Math Lab | For enrolled students only | Do not reproduce or distribute

Exercises

Work without a calculator unless a question says otherwise. In Section 8, you may use a calculator to write numbers as decimals unless the question says otherwise.

Section 1: Negative Numbers, Integers and the Number Line

1. Draw a number line to show each of the following.

(a) prime numbers > 40 but < 55 (b) integers which are at least -2

2. (a) Draw a number line to show the numbers $4, 12, 20, \ldots, 44$.

(b) Draw a number line to show all the composite numbers that are factors of both 40 and 60.

Section 2: Adding and Subtracting Integers

3. Without using a calculator, evaluate the following.

(a) $17 - 24$ (b) $-18 + 27$ (c) $-6 + 10 - 7$ (d) $-3 - 5 - 9$ (e) $25 - 31 - 7$

4. Find the missing number in each box.

(a) $5 + \square = -4$ (b) $\square - 7 = -12$ (c) $-13 + \square = -20$

(d) $12 - \square = 30$ (e) $\square - (-4) = -11$

5. Without using a calculator, evaluate the following.

(a) $7 + (-11)$ (b) $9 - (+14)$ (c) $-20 - (-6)$

(d) $-11 + (+18)$ (e) $-7 - (-16) + (-12)$

6. Evaluate each of the following without using a calculator.

(a) $6 + (-15) - 4$ (b) $-7 + (-3) - (-16)$

(c) $4 - (-11) - 8 + (-9)$ (d) $30 - (-6) + (-14) - (-7)$

(e) $-11 - [-5 + (-19)]$ (f) $[-8 + (-13)] + [16 - (-9)]$

Section 3: Multiplying and Dividing Integers; Powers and Roots

7. Without using a calculator, evaluate the following.

(a) $11 \times (-6)$ (b) $(-2) \times 4 \times (-5)$ (c) $(-6) \times (-1) \times (-9)$

(d) $(-2)^3 \times 5$ (e) $(-1)^3 \times (-15)$ (f) $(-3)^2 \times (-2)^2$

(g) $(-2) \times (-3) \times (-4) \times (-5)$

8. Evaluate each of the following without using a calculator.

(a) $14 \times (-15)$ (b) $-84 \div 7$

(c) $-96 \div (-12)$ (d) $-18 \div (-6) \times (-5)$

(e) $(-4) \times 9 \times 0 \times (-8) \times 3$ (f) $-250 \div (-5)^3 \div (-2)$

(g) $\sqrt{(-400) \div (-4)} \div (-5)$ (h) $45 \div 9 - [-12 \times (-6)]$

THE MATH LAB

© The Math Lab | For enrolled students only | Do not reproduce or distribute

Section 4: Order of Operations

9. Without using a calculator, evaluate the following. Show your working.

(a) $\dfrac{12 \times (-5) - (-9) \times 4}{(-6) \times (-2) - (-3) \times (-2)}$ (b) $2 - (-3)^3 + (2)^3 \div (-4)$

10. Without using a calculator, evaluate the following. Show your working.

(a) $(-52) - 48 \div (-4) - 5 \times (-6)$ (b) $(-3-4)^2 + [(-2)^3 + 3] - 12 \div (-3)$

11. Without using a calculator, evaluate the following. Show your working.

(a) $\{[(150 - 66) \div 7 + 38] \times 2 - 60\} \times 3$

(b) $95 - \{15 \times (-2)^2 - [3(-7) + 2^5]\}$

12. Evaluate each of the following without using a calculator.

(a) $9 + 6^2 \div 4 + \sqrt[3]{-27} \times (2 - 8)$

(b) $(-6)^2 + (-3)^3 - \left[(-31) + 45 - \sqrt[3]{125} \times (-4)\right]$

13. Fill in each box with $>$, $=$ or $<$.

(a) $-9 + (-4) \times 7 \quad \square \quad 12 \times (-4) - [-5 + (-6)]$

(b) $7 - (-15) \times 4 \div 6 \quad \square \quad [(-5) \times (-27)] \div (-10)$

(c) $\dfrac{6 \times (-5) + (-12)^2}{48 \div (-8)} \quad \square \quad \dfrac{(-2)^3 \times (-3)^2}{-20 - (-32)}$

14. Fill in each box with the correct number.

(a) $(-3) \times (\square + 4) = (-3) \times (-9) + (-3) \times 4 = \square$

(b) $(-15 - 8) \times \square = (-15) \times 3 - 8 \times 3 = \square$

(c) $5^2 + (-3)^3 = \square \div 4^2 = \square$

Section 5: Fractions and Mixed Numbers

15. Without using a calculator, simplify the following.

(a) $-\dfrac{11}{12} - \left(-\dfrac{5}{12}\right)$ (b) $-\dfrac{13}{42} + \left(-\dfrac{5}{42}\right) - \left(-\dfrac{11}{42}\right)$

16. Without using a calculator, simplify the following.

(a) $-1\dfrac{3}{4} - \left(2\dfrac{1}{3}\right)$ (b) $1\dfrac{4}{9} - 2\dfrac{5}{6}$ (c) $-3\dfrac{1}{3} + \left(-1\dfrac{1}{4}\right) - \left(-2\dfrac{1}{2}\right)$

17. Without using a calculator, simplify the following, leaving your answer as a mixed number where appropriate.

(a) $\left(-5\dfrac{1}{4}\right) \div \left(1\dfrac{3}{4}\right)$ (b) $3\dfrac{1}{3} \times \left(\dfrac{9}{20}\right) \div \left(-2\dfrac{1}{4}\right)$

18. Without using a calculator, simplify the following, leaving your answer as a mixed number where appropriate.

(a) $\left(-\dfrac{2}{3}\right) \times 1\dfrac{1}{4} + \dfrac{2}{3} \times \left(-2\dfrac{3}{4}\right)$ (b) $\left(-8\dfrac{1}{3}\right) + 2\dfrac{1}{2} \div (-6) - \dfrac{3}{8} \times \left(-2\dfrac{2}{3}\right)$

THE MATH LAB

© The Math Lab | For enrolled students only | Do not reproduce or distribute

19. Without using a calculator, simplify the following, leaving your answer as a mixed number where appropriate.

(a) $\dfrac{-1-\frac{1}{4}}{-1+\frac{3}{8}}$ (b) $2\dfrac{1}{3}-\dfrac{1-1\frac{3}{4}}{\frac{5}{6}-1\frac{1}{3}}\times 1\dfrac{2}{3}$ (c) $\dfrac{5}{12}$ of $\left(3\dfrac{1}{3}\div\dfrac{5}{9}\right)$

20. Evaluate each of the following without using a calculator.

(a) $-\dfrac{14}{15}\div\left[2\dfrac{1}{3}-\left(-1\dfrac{2}{3}\right)\right]$ (b) $-(-3)^2\times\left(\dfrac{6}{2-\frac{1}{2}}\right)\times\sqrt[3]{-8}$

21. Write each of the following as a fraction in its lowest terms.

(a) $\dfrac{1}{\sqrt{16}}-\dfrac{1}{\sqrt[3]{27}}-\left(-\dfrac{1}{\sqrt{36}}\right)$ (b) $-\sqrt{\dfrac{121}{36}}\times\sqrt[3]{\dfrac{27}{1000}}$ (c) $\dfrac{\frac{1}{\sqrt[3]{8}}\times\left(-1\frac{1}{2}\right)^2}{\frac{-\sqrt{16}}{\sqrt{9}}\div\frac{1}{\sqrt{81}}}$

22. Evaluate the following without using a calculator.

(a) $\dfrac{1}{3}\div\dfrac{5}{6}\times 4\dfrac{1}{2}$ (b) $\left(5\dfrac{5}{6}-2\dfrac{1}{3}\right)\div 1\dfrac{3}{4}$ (c) $2\dfrac{5}{8}-\left(-1\dfrac{1}{4}\right)+\sqrt{\dfrac{13}{16}-\dfrac{3}{4}}$

23. Find, in its simplest form, the number which is exactly halfway between

(a) 0.25 and $\dfrac{2}{7}$, (b) $-1\dfrac{3}{4}$ and $3\dfrac{1}{3}$, (c) 6.4 and 6.9, as a decimal.

24. (a) The number $\dfrac{7}{18}$ is exactly halfway between $\dfrac{2}{9}$ and another number, k. Find k.

(b) Write k as a decimal correct to 2 decimal places.

Section 6: Decimals

25. Evaluate each of the following without using a calculator.

(a) $-15.427-(-4.38)-6.1$ (b) -62.5×0.48 (c) $(-0.954)\div(-0.6)$

26. Evaluate each of the following without using a calculator.

(a) $-2.5\times[-4.6+(-3.28)]$ (b) $-3.5-(-0.9)^2\div(-0.3)^3$

(c) $\sqrt[3]{(-0.4)^2-0.112\div(-2)}$ (d) $\dfrac{0.48}{0.5}\times\left(\dfrac{-0.25}{1.6}\right)$

Section 7: Rational, Irrational and Real Numbers

27. Write True (T) or False (F) for each of the following statements.

(a) If the sum of two integers is negative, then both integers must be negative.

(b) Any real number multiplied by zero gives zero.

(c) Every natural number is a whole number.

(d) $\sqrt{9}$ is an irrational number.

28. Write the following rational numbers as terminating or recurring decimals.

(a) $\dfrac{9}{16}$ (b) $\dfrac{5}{6}$ (c) $-1\dfrac{18}{25}$ (d) $2\dfrac{5}{22}$

THE MATH LAB

© The Math Lab | For enrolled students only | Do not reproduce or distribute

29. Write down all the rational numbers in each of the following lists.

(a) $3.\dot{0}\dot{5}$, $\sqrt{13}$, $-2\frac{2}{3}$, π, $\frac{-7}{13}$, $\sqrt{\frac{1}{25}}$

(b) $-\sqrt{9}$, π, $\frac{13}{19}$, 1.75, $-\sqrt[3]{-64}$

(c) $2\frac{1}{5}$, -11, $\sqrt{6}$, 0.625, $\frac{\sqrt{5}}{\sqrt{9}}$

(d) $\frac{355}{113}$, $\sqrt{5}$, $\frac{\pi}{4}$, $-\frac{1}{9}$, 2.5

(e) $\left(\sqrt{7}\right)^2$, $\frac{7}{15}$, $\sqrt{11}$, -0.471, 0

30. Write down

(a) a negative rational number that is not an integer,

(b) an irrational number between 4 and 5,

(c) a whole number that is not a natural number,

(d) a rational number between $\sqrt{2}$ and $\sqrt{3}$,

(e) a perfect square between 50 and 70.

31. Copy and complete the table by putting a tick (✓) in every box that describes the number.

Number	Natural number	Whole number	Integer	Rational number	Irrational number
-6					
0					
$\frac{3}{4}$					
$\sqrt{16}$					
$\sqrt{8}$					
$0.\dot{1}$					

32. (a) Kumar says that $\sqrt{\frac{4}{9}}$ is an irrational number because it has a square root sign. Is Kumar correct? Explain your answer.

(b) Mei says that $0.1\dot{2}$ is an irrational number because its digits go on for ever. Is Mei correct? Explain your answer.

Section 8: Comparing and Ordering Real Numbers

33. Fill in each box with $>$, $=$ or $<$.

(a) $-3\frac{2}{3} \square -3.6$

(b) $-\sqrt{15} \square -\sqrt{24}$

34. Arrange the following in ascending order.

(a) $-0.\dot{3}$, $-\frac{3}{8}$, $0.\dot{3}\dot{4}$, $\frac{17}{50}$, $-0.\dot{3}0\dot{4}$

(b) $-1\frac{1}{4}$, $\frac{\pi}{3}$, $-1.\dot{2}$, $-1.\dot{2}\dot{5}$, $\frac{\sqrt{5}}{2}$

35. Arrange the following in descending order.

(a) $\frac{17}{40}$, 42%, 0.43, $\frac{11}{25}$

(b) $-2.\dot{4}1\dot{8}$, $-2.4\dot{1}\dot{8}$, $-2.41\dot{8}$, $-2\frac{5}{12}$

THE MATH LAB

© The Math Lab | For enrolled students only | Do not reproduce or distribute

36. Without using a calculator, write down the two consecutive integers (integers next to each other) between which each number lies.

(a) $\sqrt{45}$ (b) $-\sqrt{70}$ (c) $\sqrt[3]{90}$ (d) $-\sqrt[3]{40}$

(e) Hence arrange the four numbers in ascending order.

Section 9: Using a Calculator

37. Use a calculator to evaluate each of the following, giving your answers correct to 2 decimal places.

(a) $\dfrac{(0.48+3.95)^5-2\frac{1}{3}}{4.6^3}$ (b) $\dfrac{\left(4\frac{5}{9}\right)^3-\left(-2\frac{3}{8}\right)^2}{-\sqrt{\frac{49}{64}}\div\sqrt[3]{-\frac{1}{8}}}$

(c) $\left(-\dfrac{7}{8}\right)^2\times(-\pi)+\sqrt[3]{53}\div 4\times\left(-\dfrac{\pi}{3}\right)$ (d) $\sqrt[3]{\dfrac{3\frac{2}{13}+\left(-6\frac{1}{3}\right)}{0.083^3}}$

38. Use a calculator to evaluate each of the following, giving your answers correct to 3 decimal places.

(a) $\dfrac{-\frac{7}{23}+\frac{13}{17}}{-3\frac{11}{19}-\left(-\frac{5}{9}\right)^2}$ (b) $\dfrac{\sqrt{42}-\sqrt[3]{33}}{-\pi+\sqrt{\left[-2\frac{1}{2}\times\left(-\frac{4}{5}\right)\right]}}$

(c) $\dfrac{\frac{13}{19}\div\left[\left(1\frac{7}{8}-3\right)^3-\frac{23}{6}\right]}{\sqrt{\pi}+\left(2\frac{3}{4}\right)^2}$ (d) $\sqrt[3]{\dfrac{(-3)^3-(-4)^2\times(-7)^2}{12^2+97-\left(-\frac{\pi}{3}\right)}}$

(e) $\left\{\left(\dfrac{1}{3}\right)^2-\sqrt{\dfrac{5}{36}}\times\left[-\sqrt{\dfrac{3}{7}}-(-0.25)^3\right]\right\}\times[-3.1\div(-\pi)]$

Section 10: Real-World Problems

39. The temperature inside a freezer is $-21\,^\circ\text{C}$. During a power cut, the temperature inside the freezer rises by $2.5\,^\circ\text{C}$ every hour.

(a) Find the temperature inside the freezer 4 hours after the power cut starts.

(b) After how many whole hours does the temperature first rise above $-4\,^\circ\text{C}$?

40. A food stall writes down its profit for each week as a positive number of dollars, and its loss as a negative number of dollars. The table shows its records for four weeks.

Week	1	2	3	4
Profit ($)	96	−153	118	−74

(a) Find the total profit or loss over the four weeks.

(b) Find the difference between the stall's profit in its best week and its profit in its worst week.

41. The table below shows the temperatures in four cities at noon on a particular day.

Bangkok	Chicago	Berlin	Warsaw
$33\,^\circ\text{C}$	$-10\,^\circ\text{C}$	$2\,^\circ\text{C}$	$-3.5\,^\circ\text{C}$

(a) (i) Which city was $12\,^\circ\text{C}$ warmer than Chicago?

(ii) Which two cities had temperatures $5.5\,^\circ\text{C}$ apart?

(b) At the same time, the temperature in Madrid was exactly midway between the temperatures in Bangkok and Chicago. What was the temperature in Madrid?

THE MATH LAB

© The Math Lab | For enrolled students only | Do not reproduce or distribute

42. A submarine is at $-315\,\text{m}$, that is, $315\,\text{m}$ below sea level. It rises at a steady rate of $24\,\text{m}$ per minute.

(a) Find the position of the submarine 7 minutes later.

(b) How many minutes does the submarine take to rise from $-315\,\text{m}$ to $-51\,\text{m}$?

43. On a cold day, the temperature at the top of a mountain $1600\,\text{m}$ above a village is $-4\,^\circ\text{C}$, and the temperature on the valley floor $800\,\text{m}$ below the village is $14\,^\circ\text{C}$. Assume that the temperature changes at a steady rate with height. Find

(a) the difference between these two temperatures,

(b) the temperature at the village.

44. (a) A diver is $23\,\text{m}$ below the surface of the sea, while a seagull flies $41\,\text{m}$ above the sea, directly above the diver. What is the vertical distance between the diver and the seagull?

(b) A drone is flying $78\,\text{m}$ above sea level. A submarine is directly below the drone, $205\,\text{m}$ away from it. How far below sea level is the submarine?

45. In a building, the ground floor is level 0, the floors above it are levels $1, 2, 3, \ldots$ and the basement floors are levels $-1, -2, -3, \ldots$ A lift starts at level 7. It goes up 4 levels, down 13 levels, up 10 levels and then down 11 levels.

(a) At which level does the lift stop?

(b) The lift is then called to level 12. How many levels must it go up?

46. A tank is full of water. On Monday, $\frac{1}{3}$ of the water is used. On Tuesday, $\frac{3}{10}$ of the remaining water is used. Altogether, 320 litres of water have been used. How many litres of water are left in the tank?

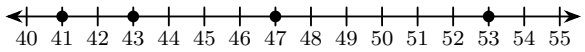
THE MATH LAB

© The Math Lab | For enrolled students only | Do not reproduce or distribute

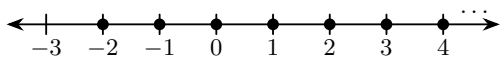
Solutions to Exercises

Section 1 — Negative Numbers, Integers and the Number Line (Questions 1–2)

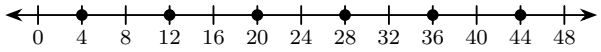
1(a) The prime numbers between 40 and 55 are $41, 43, 47, 53$.



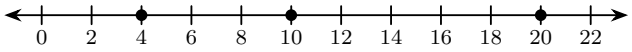
1(b) $-2, -1, 0, 1, 2, \ldots$ (the integers carry on without end).



2(a) $4, 12, 20, 28, 36, 44$



2(b) Composite numbers are positive, so use the positive factors. Factors of 40: $1, 2, 4, 5, 8, 10, 20, 40$. Factors of 60: $1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60$. Common factors: $1, 2, 4, 5, 10, 20$; the composite ones are $4, 10, 20$.



Section 2 — Adding and Subtracting Integers (Questions 3–6)

3 (a) $\boxed{-7}$ (b) $\boxed{9}$ (c) $4 - 7 = \boxed{-3}$ (d) $-8 - 9 = \boxed{-17}$
(e) $-6 - 7 = \boxed{-13}$

4 (a) $\square = -4 - 5 = \boxed{-9}$ (b) $\square = -12 + 7 = \boxed{-5}$
(c) $\square = -20 - (-13) = \boxed{-7}$ (d) $\square = 12 - 30 = \boxed{-18}$
(e) $\square + 4 = -11$, $\square = -11 - 4 = \boxed{-15}$

5 (a) $7 - 11 = \boxed{-4}$ (b) $9 - 14 = \boxed{-5}$
(c) $-20 + 6 = \boxed{-14}$ (d) $-11 + 18 = \boxed{7}$
(e) $-7 + 16 - 12 = \boxed{-3}$

6(a) $6 + (-15) - 4 = 6 - 15 - 4 = \boxed{-13}$

6(b) $-7 + (-3) - (-16) = -7 - 3 + 16 = \boxed{6}$

6(c) $4 - (-11) - 8 + (-9) = 4 + 11 - 8 - 9 = \boxed{-2}$

6(d) $30 - (-6) + (-14) - (-7) = 30 + 6 - 14 + 7 = \boxed{29}$

6(e) $-11 - [-5 + (-19)] = -11 - (-24) = -11 + 24 = \boxed{13}$

6(f) $[-8 + (-13)] + [16 - (-9)] = (-21) + 25 = \boxed{4}$

Section 3 — Multiplying and Dividing Integers; Powers and Roots (Questions 7–8)

7 (a) $\boxed{-66}$ (b) $(-8) \times (-5) = \boxed{40}$ (c) $6 \times (-9) = \boxed{-54}$
(d) $(-8) \times 5 = \boxed{-40}$ (e) $(-1) \times (-15) = \boxed{15}$
(f) $9 \times 4 = \boxed{36}$ (g) $6 \times 20 = \boxed{120}$

8(a)–(c) (a) $14 \times (-15) = \boxed{-210}$ (b) $-84 \div 7 = \boxed{-12}$
(c) $-96 \div (-12) = \boxed{8}$

8(d) $-18 \div (-6) \times (-5) = 3 \times (-5) = \boxed{-15}$

8(e) One of the factors is 0, so the product is $\boxed{0}$.

8(f) $-250 \div (-5)^3 \div (-2) = -250 \div (-125) \div (-2) = 2 \div (-2) = \boxed{-1}$

8(g) $\sqrt{(-400) \div (-4)} \div (-5) = \sqrt{100} \div (-5) = 10 \div (-5) = \boxed{-2}$

8(h) $45 \div 9 - [-12 \times (-6)] = 5 - 72 = \boxed{-67}$

Section 4 — Order of Operations (Questions 9–14)

9(a) Numerator
$= 12 \times (-5) - (-9) \times 4 = -60 - (-36) = -24$;
denominator $= (-6) \times (-2) - (-3) \times (-2) = 12 - 6 = 6$.
$\dfrac{-24}{6} = \boxed{-4}$

9(b) $2 - (-3)^3 + (2)^3 \div (-4) = 2 - (-27) + 8 \div (-4)$
$= 2 + 27 + (-2) = \boxed{27}$

10(a) $(-52) - 48 \div (-4) - 5 \times (-6) = -52 - (-12) - (-30)$
$= -52 + 12 + 30 = \boxed{-10}$

10(b) $(-3 - 4)^2 + [(-2)^3 + 3] - 12 \div (-3)$
$= (-7)^2 + (-8 + 3) - (-4) = 49 - 5 + 4 = \boxed{48}$

11(a) $\{[(150 - 66) \div 7 + 38] \times 2 - 60\} \times 3$
$= \{[84 \div 7 + 38] \times 2 - 60\} \times 3 = \{[12 + 38] \times 2 - 60\} \times 3$
$= \{100 - 60\} \times 3 = 40 \times 3 = \boxed{120}$

11(b) $95 - \{15 \times (-2)^2 - [3(-7) + 2^5]\}$
$= 95 - \{15 \times 4 - [-21 + 32]\} = 95 - \{60 - 11\}$
$= 95 - 49 = \boxed{46}$

12(a) $9 + 6^2 \div 4 + \sqrt[3]{-27} \times (2 - 8) = 9 + 36 \div 4 + (-3) \times (-6)$
$= 9 + 9 + 18 = \boxed{36}$

12(b) $(-6)^2 + (-3)^3 - \left[(-31) + 45 - \sqrt[3]{125} \times (-4)\right]$
$= 36 - 27 - [-31 + 45 - 5 \times (-4)] = 9 - [14 + 20]$
$= 9 - 34 = \boxed{-25}$

13(a) Left side $= -9 + (-28) = -37$; right side
$= -48 - (-11) = -48 + 11 = -37$.
$-37 \boxed{=} -37$

THE MATH LAB

© The Math Lab | For enrolled students only | Do not reproduce or distribute

13(b) Left side $= 7-(-60)\div 6 = 7+10 = 17$; right side $= 135\div(-10) = -13.5$.
$17\ \boxed{>}\ -13.5$

13(c) Left side $= \dfrac{-30+144}{-6} = \dfrac{114}{-6} = -19$;
right side $= \dfrac{(-8)\times 9}{-20+32} = \dfrac{-72}{12} = -6$.
$-19\ \boxed{<}\ -6$

14(a) Each number in the bracket is multiplied by -3, so the first box is -9:
$(-3)\times(-9)+(-3)\times 4 = 27-12 = 15$.
Boxes: $\boxed{-9}$ and $\boxed{15}$

14(b) Each number in the bracket is multiplied by the same number, 3: $(-15)\times 3-8\times 3 = -45-24 = -69$.
Boxes: $\boxed{3}$ and $\boxed{-69}$

14(c) $5^2+(-3)^3 = 25+(-27) = -2$, and
$-2 = (-32)\div 16 = (-32)\div 4^2$.
Boxes: $\boxed{-32}$ and $\boxed{-2}$

Section 5 — Fractions and Mixed Numbers (Questions 15–24)

15(a) $\dfrac{-11+5}{12} = -\dfrac{6}{12} = \boxed{-\dfrac{1}{2}}$

15(b) $\dfrac{-13-5+11}{42} = -\dfrac{7}{42} = \boxed{-\dfrac{1}{6}}$

16(a) $-\dfrac{7}{4}-\dfrac{7}{3} = -\dfrac{21}{12}-\dfrac{28}{12} = -\dfrac{49}{12} = \boxed{-4\dfrac{1}{12}}$

16(b) $\dfrac{13}{9}-\dfrac{17}{6} = \dfrac{26}{18}-\dfrac{51}{18} = -\dfrac{25}{18} = \boxed{-1\dfrac{7}{18}}$

16(c) $-\dfrac{10}{3}-\dfrac{5}{4}+\dfrac{5}{2} = \dfrac{-40-15+30}{12} = -\dfrac{25}{12} = \boxed{-2\dfrac{1}{12}}$

17(a) $-\dfrac{21}{4}\div\dfrac{7}{4} = -\dfrac{\cancel{21}^{3}}{\cancel{4}_{1}}\times\dfrac{\cancel{4}^{1}}{\cancel{7}_{1}} = \boxed{-3}$

17(b) $\dfrac{10}{3}\times\dfrac{9}{20} = \dfrac{3}{2}$; $\dfrac{3}{2}\div\left(-\dfrac{9}{4}\right) = \dfrac{3}{2}\times\left(-\dfrac{4}{9}\right) = \boxed{-\dfrac{2}{3}}$

18(a) $-\dfrac{2}{3}\times\dfrac{5}{4}+\dfrac{2}{3}\times\left(-\dfrac{11}{4}\right) = -\dfrac{10}{12}-\dfrac{22}{12} = -\dfrac{32}{12} = -\dfrac{8}{3} = \boxed{-2\dfrac{2}{3}}$

18(b) $-\dfrac{25}{3}+\dfrac{5}{2}\times\left(-\dfrac{1}{6}\right)-\dfrac{3}{8}\times\left(-\dfrac{8}{3}\right) = -\dfrac{25}{3}-\dfrac{5}{12}+1 = \dfrac{-100-5+12}{12} = -\dfrac{93}{12} = -\dfrac{31}{4} = \boxed{-7\dfrac{3}{4}}$

19(a) $\dfrac{-\frac{5}{4}}{-\frac{5}{8}} = \dfrac{5}{4}\times\dfrac{8}{5} = \boxed{2}$

19(b) $1-1\dfrac{3}{4} = -\dfrac{3}{4}$ and $\dfrac{5}{6}-1\dfrac{1}{3} = \dfrac{5}{6}-\dfrac{8}{6} = -\dfrac{1}{2}$, so the fraction $= \left(-\dfrac{3}{4}\right)\div\left(-\dfrac{1}{2}\right) = \dfrac{3}{2}$.
$2\dfrac{1}{3}-\dfrac{3}{2}\times\dfrac{5}{3} = 2\dfrac{1}{3}-\dfrac{5}{2} = \dfrac{14}{6}-\dfrac{15}{6} = \boxed{-\dfrac{1}{6}}$

19(c) $3\dfrac{1}{3}\div\dfrac{5}{9} = \dfrac{10}{3}\times\dfrac{9}{5} = 6$; $\dfrac{5}{12}\times 6 = \dfrac{5}{2} = \boxed{2\dfrac{1}{2}}$

20(a) $2\dfrac{1}{3}+1\dfrac{2}{3} = 4$; $-\dfrac{14}{15}\div 4 = -\dfrac{14}{60} = \boxed{-\dfrac{7}{30}}$

20(b) $\dfrac{6}{2-\frac{1}{2}} = 6\div\dfrac{3}{2} = 4$; $-9\times 4\times(-2) = \boxed{72}$

21(a) $\dfrac{1}{4}-\dfrac{1}{3}+\dfrac{1}{6} = \dfrac{3-4+2}{12} = \boxed{\dfrac{1}{12}}$

21(b) $-\dfrac{11}{6}\times\dfrac{3}{10} = -\dfrac{33}{60} = \boxed{-\dfrac{11}{20}}$

21(c) Numerator $= \dfrac{1}{2}\times\left(-\dfrac{3}{2}\right)^2 = \dfrac{1}{2}\times\dfrac{9}{4} = \dfrac{9}{8}$;
denominator $= -\dfrac{4}{3}\div\dfrac{1}{9} = -\dfrac{4}{3}\times 9 = -12$.
$\dfrac{9}{8}\times\left(-\dfrac{1}{12}\right) = -\dfrac{9}{96} = \boxed{-\dfrac{3}{32}}$

22(a) $\dfrac{1}{3}\times\dfrac{6}{5}\times\dfrac{9}{2} = \dfrac{54}{30} = \dfrac{9}{5} = \boxed{1\dfrac{4}{5}}$

22(b) $\left(\dfrac{35}{6}-\dfrac{14}{6}\right)\div\dfrac{7}{4} = \dfrac{21}{6}\times\dfrac{4}{7} = \boxed{2}$

22(c) $\sqrt{\dfrac{13}{16}-\dfrac{3}{4}} = \sqrt{\dfrac{13-12}{16}} = \dfrac{1}{4}$;
$\dfrac{21}{8}+\dfrac{5}{4}+\dfrac{1}{4} = \dfrac{21+10+2}{8} = \dfrac{33}{8} = \boxed{4\dfrac{1}{8}}$

23(a) $\dfrac{\frac{1}{4}+\frac{2}{7}}{2} = \dfrac{15}{28}\times\dfrac{1}{2} = \boxed{\dfrac{15}{56}}$

23(b) $\dfrac{-\frac{7}{4}+\frac{10}{3}}{2} = \dfrac{-21+40}{12}\times\dfrac{1}{2} = \boxed{\dfrac{19}{24}}$

23(c) $\dfrac{6.4+6.9}{2} = \dfrac{13.3}{2} = \boxed{6.65}$

24(a) The two numbers add up to twice the number halfway between them: $\dfrac{2}{9}+k = 2\times\dfrac{7}{18} = \dfrac{7}{9}$.
$k = \dfrac{7}{9}-\dfrac{2}{9} = \boxed{\dfrac{5}{9}}$

24(b) $\dfrac{5}{9} = 0.555\,55\ldots = \boxed{0.56}$ (to 2 d.p.)

Section 6 — Decimals (Questions 25–26)

25(a) $-15.427+4.38-6.1 = -11.047-6.1 = \boxed{-17.147}$

25(b) $625\times 48 = 30\,000$; $1+2 = 3$ d.p.:
$-62.5\times 0.48 = \boxed{-30}$

THE MATH LAB

© The Math Lab | For enrolled students only | Do not reproduce or distribute

25(c) $\dfrac{-0.954}{-0.6} = \dfrac{9.54}{6} = \boxed{1.59}$

26(a) $-2.5 \times (-4.6 - 3.28) = -2.5 \times (-7.88) = \boxed{19.7}$

26(b) $(-0.9)^2 = 0.81$, $(-0.3)^3 = -0.027$,
$0.81 \div (-0.027) = -\dfrac{810}{27} = -30$;
$-3.5 - (-30) = -3.5 + 30 = \boxed{26.5}$

26(c) $\sqrt[3]{0.16 - (-0.056)} = \sqrt[3]{0.16 + 0.056} = \sqrt[3]{0.216} = \boxed{0.6}$

26(d) $\dfrac{0.48}{1.6} \times \left(-\dfrac{0.25}{0.5}\right) = 0.3 \times (-0.5) = \boxed{-0.15}$

Section 7 — Rational, Irrational and Real Numbers (Questions 27–32)

27 (a) $\boxed{\text{F}}$ (b) $\boxed{\text{T}}$ (c) $\boxed{\text{T}}$ (d) $\boxed{\text{F}}$
For (a): $3 + (-5) = -2$ is negative, but 3 is positive.
For (d): $\sqrt{9} = 3$.

28 (a) $\boxed{0.5625}$ (b) $\boxed{0.8\dot{3}}$ (c) $\boxed{-1.72}$ (d) $\boxed{2.2\dot{2}\dot{7}}$

29(a) $\boxed{3.\dot{0}\dot{5},\ -2\dfrac{2}{3},\ \dfrac{-7}{13},\ \sqrt{\dfrac{1}{25}}}$ $\left(\sqrt{\frac{1}{25}} = \frac{1}{5}\right)$

29(b) $\boxed{-\sqrt{9},\ \dfrac{13}{19},\ 1.75,\ -\sqrt[3]{-64}}$ $(-\sqrt{9} = -3,\ -\sqrt[3]{-64} = 4)$

29(c) $\boxed{2\dfrac{1}{5},\ -11,\ 0.625}$ $\left(\frac{\sqrt{5}}{\sqrt{9}} = \frac{\sqrt{5}}{3}\right.$ is irrational$)$

29(d) $\boxed{\dfrac{355}{113},\ -\dfrac{1}{9},\ 2.5}$

29(e) $\boxed{\left(\sqrt{7}\right)^2,\ \dfrac{7}{15},\ -0.471,\ 0}$
$\left(\left(\sqrt{7}\right)^2 = \sqrt{7} \times \sqrt{7} = 7\right)$

30(a) Any negative fraction, or negative terminating or recurring decimal, that is not an integer; other answers are possible, e.g. $\boxed{-\dfrac{1}{2}}$

30(b) $4^2 = 16$ and $5^2 = 25$, so $\sqrt{17}, \sqrt{18}, \ldots, \sqrt{24}$ all lie between 4 and 5, e.g. $\boxed{\sqrt{17}}$

30(c) 0 is the only one: $\boxed{0}$

30(d) $\sqrt{2} = 1.414\ldots$ and $\sqrt{3} = 1.732\ldots$, e.g. $\boxed{1.5}$

30(e) $7^2 = 49$, $8^2 = 64$ and $9^2 = 81$, so the only one is $\boxed{64}$

31 $\sqrt{16} = 4$; $\sqrt{8}$ is irrational (8 is not a perfect square).

Number	Natural	Whole	Integer	Rational	Irrational
-6			✓	✓	
0		✓	✓	✓	
$\frac{3}{4}$				✓	
$\sqrt{16}$	✓	✓	✓	✓	
$\sqrt{8}$					✓
$0.\dot{1}$				✓	

32(a) $\boxed{\text{No}}$ $\sqrt{\frac{4}{9}} = \frac{2}{3}$, which is one integer divided by another, so it is rational.

32(b) $\boxed{\text{No}}$ $0.1\dot{2} = 0.1222\ldots$ is a recurring decimal (the 2 repeats), so it is rational.

Section 8 — Comparing and Ordering Real Numbers (Questions 33–36)

33 (a) $\boxed{<}$ (b) $\boxed{>}$
For (a): $-3\frac{2}{3} = -3.666\ldots$ lies to the left of -3.6. For (b): $\sqrt{15} < \sqrt{24}$, so $-\sqrt{15} > -\sqrt{24}$.

34(a) $-0.333\,33$, $-0.375\,00$, $0.343\,43$, $0.340\,00$, $-0.304\,30$
$\boxed{-\dfrac{3}{8},\ -0.\dot{3},\ -0.\dot{3}0\dot{4},\ \dfrac{17}{50},\ 0.\dot{3}\dot{4}}$

34(b) -1.25, $1.047\,20$, $-1.222\,22$, $-1.252\,53$, $1.118\,03$
$\boxed{-1.\dot{2}\dot{5},\ -1\dfrac{1}{4},\ -1.\dot{2},\ \dfrac{\pi}{3},\ \dfrac{\sqrt{5}}{2}}$

35(a) 0.425, 0.42, 0.43, 0.44 $\boxed{\dfrac{11}{25},\ 0.43,\ \dfrac{17}{40},\ 42\%}$

35(b) $-2.418\,42$, $-2.418\,18$, $-2.418\,89$, $-2.416\,67$
$\boxed{-2\dfrac{5}{12},\ -2.4\dot{1}\dot{8},\ -2.\dot{4}1\dot{8},\ -2.41\dot{8}}$

36(a) $6^2 = 36 < 45 < 49 = 7^2$, so $6 < \sqrt{45} < 7$:
$\boxed{6 \text{ and } 7}$

36(b) $8^2 = 64 < 70 < 81 = 9^2$, so $8 < \sqrt{70} < 9$ and $-9 < -\sqrt{70} < -8$: $\boxed{-9 \text{ and } -8}$

36(c) $4^3 = 64 < 90 < 125 = 5^3$, so $4 < \sqrt[3]{90} < 5$:
$\boxed{4 \text{ and } 5}$

36(d) $3^3 = 27 < 40 < 64 = 4^3$, so $3 < \sqrt[3]{40} < 4$ and $-4 < -\sqrt[3]{40} < -3$: $\boxed{-4 \text{ and } -3}$

36(e) $\boxed{-\sqrt{70},\ -\sqrt[3]{40},\ \sqrt[3]{90},\ \sqrt{45}}$

THE MATH LAB

© The Math Lab | For enrolled students only | Do not reproduce or distribute

Section 9 — Using a Calculator (Questions 37–38)

37 Each value is the calculator display, rounded to 2 decimal places.

(a) $17.504\,543\,5\ldots = \boxed{17.50}$ (b) $50.800\,693\,2\ldots = \boxed{50.80}$

(c) $-3.388\,675\,18\ldots = \boxed{-3.39}$

(d) $-17.716\,350\,6\ldots = \boxed{-17.72}$

38 Each value is the calculator display, rounded to 3 decimal places.

(a) $-0.118\,417\,357\ldots = \boxed{-0.118}$

(b) $-1.894\,897\,52\ldots = \boxed{-1.895}$

(c) $-0.013\,942\,037\,9\ldots = \boxed{-0.014}$

(d) $-1.496\,374\,38\ldots = \boxed{-1.496}$

(e) $0.344\,639\,019\ldots = \boxed{0.345}$

Section 10 — Real-World Problems (Questions 39–46)

39(a) Rise in 4 hours $= 4 \times 2.5 = 10\,^\circ\text{C}$.

$-21\,^\circ\text{C} + 10\,^\circ\text{C} = \boxed{-11\,^\circ\text{C}}$

39(b) The temperature must rise by more than $-4 - (-21) = 17\,^\circ\text{C}$; $17 \div 2.5 = 6.8$

After 6 hours: $-21 + 6 \times 2.5 = -6\,^\circ\text{C}$, which is not above $-4\,^\circ\text{C}$.

After 7 hours: $-21 + 7 \times 2.5 = -3.5\,^\circ\text{C}$, which is above $-4\,^\circ\text{C}$. $\boxed{\text{7 hours}}$

40(a) $96 - 153 + 118 - 74 = -13$: a total $\boxed{\text{loss of \$13}}$

40(b) Profit in the best week (week 3) = \$118; in the worst week (week 2) = −\$153.

$118 - (-153) = 118 + 153 = \boxed{\$271}$

41(a)(i) $-10\,^\circ\text{C} + 12\,^\circ\text{C} = 2\,^\circ\text{C}$: $\boxed{\text{Berlin}}$

41(a)(ii) $2 - (-3.5) = 2 + 3.5 = 5.5\,^\circ\text{C}$ (no other pair is $5.5\,^\circ\text{C}$ apart): $\boxed{\text{Berlin and Warsaw}}$

41(b) $\dfrac{33 + (-10)}{2} = \dfrac{23}{2} = \boxed{11.5\,^\circ\text{C}}$

42(a) In 7 minutes it rises $7 \times 24 = 168\,\text{m}$.

$-315 + 168 = \boxed{-147\,\text{m}}$, that is, 147 m below sea level.

42(b) Distance risen

$= -51 - (-315) = -51 + 315 = 264\,\text{m}$.

$264 \div 24 = \boxed{\text{11 minutes}}$

43(a) $14 - (-4) = 14 + 4 = \boxed{18\,^\circ\text{C}}$

43(b) The mountain top and the valley floor are $1600 + 800 = 2400\,\text{m}$ apart, and the temperature falls as we go up.

$2400\,\text{m} \to 18\,^\circ\text{C}$; $800\,\text{m} \to \dfrac{18}{2400} \times 800 = 6\,^\circ\text{C}$.

The village is 800 m above the valley floor:

$14 - 6 = \boxed{8\,^\circ\text{C}}$

44(a) $41 - (-23) = 41 + 23 = \boxed{64\,\text{m}}$

44(b) The submarine is 205 m below the drone:

$78 - 205 = -127$. $\boxed{127\,\text{m}}$ below sea level

45(a) $7 + 4 - 13 + 10 - 11 = -3$: $\boxed{\text{level } -3}$

45(b) $12 - (-3) = 12 + 3 = \boxed{15\text{ levels}}$

46 Tuesday $= \frac{3}{10} \times \left(1 - \frac{1}{3}\right) = \frac{3}{10} \times \frac{2}{3} = \frac{1}{5}$ of the tank.

Used $= \frac{1}{3} + \frac{1}{5} = \frac{8}{15}$ of the tank $= 320$ litres.

Left $= \frac{7}{15}$ of the tank $= \frac{320}{8} \times 7 = \boxed{280\text{ litres}}$

THE MATH LAB

© The Math Lab | For enrolled students only | Do not reproduce or distribute