Prime Numbers, HCF and LCM

Prime Factors, Squares and Cubes, Highest Common Factor and Lowest Common Multiple

Secondary 1 — G3 Mathematics

Notes

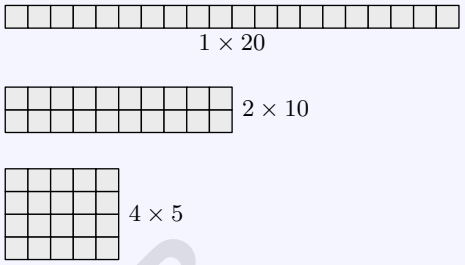
Whole Numbers, Factors and Multiples

The **whole numbers** are $0, 1, 2, 3, 4, 5, \ldots$ A whole number that can be divided exactly by 2 is **even** $(0, 2, 4, 6, \ldots)$; one that cannot is **odd** $(1, 3, 5, 7, \ldots)$.

A **factor** of a number divides it exactly, leaving no remainder. Factors come in pairs that multiply to give the number:

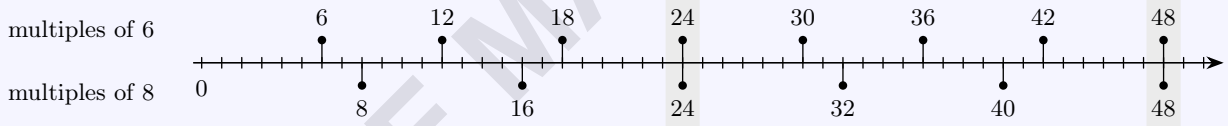
$$20 = 1 \times 20 = 2 \times 10 = 4 \times 5,$$

so the factors of 20 are $1, 2, 4, 5, 10$ and 20. Each pair matches one way of arranging 20 unit squares into a rectangle.



Factors that two numbers share are their **common factors**. The factors of 16 are $1, 2, 4, 8, 16$ and the factors of 28 are $1, 2, 4, 7, 14, 28$, so the common factors of 16 and 28 are 1, 2 and 4.

A **multiple** of a number is the number multiplied by $1, 2, 3, \ldots$: the multiples of 6 are $6, 12, 18, 24, \ldots$ and the multiples of 8 are $8, 16, 24, 32, \ldots$ A **common multiple** of two numbers is a multiple of both. The first two common multiples of 6 and 8 are 24 and 48:

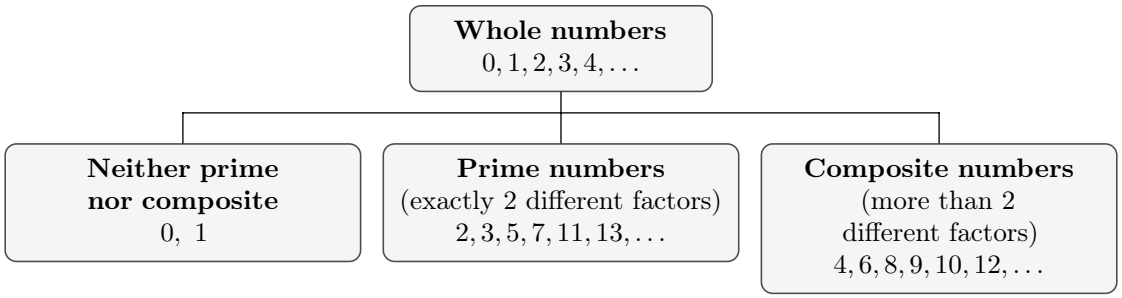


Prime Numbers and Composite Numbers

A **prime number** is a whole number with exactly two different factors: 1 and the number itself. For example, the only factors of 7 are 1 and 7, so 7 is prime. The prime numbers begin $2, 3, 5, 7, 11, 13, 17, 19, \ldots$ Notice that 2 is the only even prime number.

A **composite number** is a whole number with more than two different factors. For example, $10 = 1 \times 10 = 2 \times 5$ has the four factors $1, 2, 5$ and 10, so 10 is composite. Every whole number greater than 1 is either prime or composite.

The numbers 0 and 1 are neither prime nor composite.



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1	**2**	**3**	4	**5**	6	**7**	8	9	10
11	12	**13**	14	15	16	**17**	18	**19**	20
21	22	**23**	24	25	26	27	28	**29**	30
31	32	33	34	35	36	**37**	38	39	40
41	42	**43**	44	45	46	**47**	48	49	50
51	52	**53**	54	55	56	57	58	**59**	60
61	62	63	64	65	66	**67**	68	69	70
71	72	**73**	74	75	76	77	78	**79**	80
81	82	**83**	84	85	86	87	88	**89**	90
91	92	93	94	95	96	**97**	98	99	100

The 25 prime numbers below 100 are shaded.

Is a whole number $n > 1$ prime?
Divide n by each prime number up to $\sqrt{n}$.

(a) If one of these primes divides n exactly, n is **composite**.

(b) If none of them divides n exactly, n is **prime**.

Why can we stop at $\sqrt{n}$? If $n = a \times b$, then a and b cannot both be bigger than $\sqrt{n}$, or their product would be bigger than n. So a composite number n always has a factor no bigger than $\sqrt{n}$, and the prime factors of that factor are no bigger than $\sqrt{n}$ either. One of the primes we try would then divide n exactly.

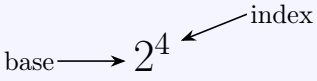
A quick test for 3. A number can be divided exactly by 3 when the sum of its digits can be divided exactly by 3. For 852, the digit sum is $8 + 5 + 2 = 15$, and $15 \div 3 = 5$, so 852 can be divided exactly by 3 ($852 = 3 \times 284$).

Prime Factors and Prime Factorisation

A **prime factor** of a number is a factor that is also a prime number. The factors of 20 are $1, 2, 4, 5, 10$ and 20; only 2 and 5 are prime, so the prime factors of 20 are 2 and 5.

Writing a composite number as a product of prime numbers is called **prime factorisation**. Two common methods are the **factor tree** and **repeated division**; both give the same answer.

Index notation. A factor that is repeated is written as a power: $2 \times 2 \times 2 \times 2 = 2^4$, read as "2 to the power of 4". The number being multiplied, 2, is the **base**; the number of times it appears, 4, is the **index**. In the same way, $3 \times 3 \times 5 \times 5 \times 5 = 3^2 \times 5^3$.



Two Ways to Find the Prime Factors

Factor tree.

1. Put the number at the top of the tree.
2. Split it into two factors.
3. Keep splitting any factor that is not prime.
4. Stop when every branch ends in a prime number. Circle the primes: their product is the prime factorisation.

Repeated division.

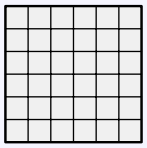
1. Divide by the smallest prime that goes into the number exactly.
2. Divide the answer again, by the same prime or a bigger one, until you reach 1.
3. Multiply all the primes you divided by, and write the product in index notation.

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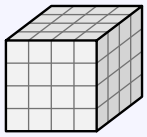
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Squares, Cubes and Their Roots

Multiplying a number by itself gives its **square**: $6 \times 6 = 6^2 = 36$, read as "6 squared". Going back the other way, 6 is the **square root** of 36, written $\sqrt{36} = 6$; the sign $\sqrt{\ }$ always means the positive square root. A whole number whose square root is a whole number is a **perfect square** (or **square number**): $1, 4, 9, 16, 25, 36, \ldots$



$6^2 = 36$



$4^3 = 64$

Multiplying a number by itself three times gives its **cube**: $4 \times 4 \times 4 = 4^3 = 64$, read as "4 cubed". 4 is the **cube root** of 64, written $\sqrt[3]{64} = 4$. A whole number whose cube root is a whole number is a **perfect cube** (or **cube number**): $1, 8, 27, 64, 125, \ldots$

To find the square root or cube root of a fraction, take the root of the top and of the bottom: $\sqrt{\frac{9}{25}} = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5}$ and $\sqrt[3]{\frac{8}{125}} = \frac{\sqrt[3]{8}}{\sqrt[3]{125}} = \frac{2}{5}$.

Using Prime Factors to Spot Squares and Cubes

Write the number in index notation using its prime factors. The number is

- a **perfect square** when every index is even (a multiple of 2): the prime factors can be shared out into 2 identical groups;
- a **perfect cube** when every index is a multiple of 3: the prime factors can be shared out into 3 identical groups;
- **both** a perfect square and a perfect cube when every index is a multiple of 6.

Making a perfect square or cube. To find the smallest positive whole number k that makes $N \times k$ a perfect square (or cube), let k bring in only the missing prime factors, so that each index goes *up* to the nearest multiple of 2 (or 3). To make $N \div k$ a perfect square (or cube), let k take away only the extra prime factors, so that each index comes *down* to the nearest multiple of 2 (or 3).

Once you have the smallest k that makes $N \times k$ a perfect square, every other k that works is that smallest k multiplied by a perfect square $(4, 9, 16, \ldots)$.

Finding a root. Write the number in index notation using its prime factors. For a square root, share the prime factors out into 2 identical groups; for a cube root, into 3 identical groups. The root is the product of the factors in one group.

Checking. After finding k, multiply (or divide) to check that the result really is a perfect square or a perfect cube.

Multiplying by $\frac{p}{q}$, where p and q are primes. Multiplying by p brings in exactly one prime factor, and dividing by q takes out exactly one. For each prime, decide whether one more factor or one fewer will make its index a multiple of 3 (for a cube) or of 2 (for a square).

HCF and LCM

The **highest common factor (HCF)** of two or more numbers is the largest number that is a factor of all of them. The factors of 16 are $1, 2, 4, 8, 16$ and the factors of 24 are $1, 2, 3, 4, 6, 8, 12, 24$. The common factors are $1, 2, 4, 8$, so the HCF of 16 and 24 is 8.

The **lowest common multiple (LCM)** of two or more numbers is the smallest positive number that is a multiple of all of them. The multiples of 6 are $6, 12, 18, 24, 30, 36, \ldots$ and the multiples of 9 are $9, 18, 27, 36, \ldots$ The common multiples are $18, 36, \ldots$, so the LCM of 6 and 9 is 18.

The common factors $(1, 2, 4, 8)$ are exactly the factors of the HCF, and the common multiples $(18, 36, 54, \ldots)$ are exactly the multiples of the LCM.

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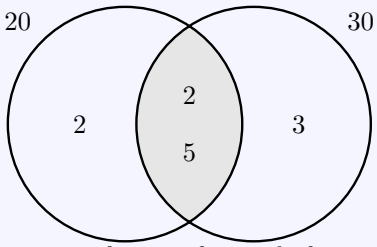
Finding the HCF and LCM from Prime Factors

Write each number in index notation using its prime factors. Then

- **HCF:** use only the primes found in *every* number, each with its *smallest* index;
- **LCM:** use *every* prime that appears, each with its *largest* index.

For $20 = 2^2 \times 5$ and $30 = 2 \times 3 \times 5$, the prime factors the two numbers share sit where the circles overlap:

$$\text{HCF} = 2 \times 5 = 10, \qquad \text{LCM} = 2^2 \times 3 \times 5 = 60.$$



prime factors of 20 and of 30
HCF: multiply the overlap
LCM: multiply everything

Finding the HCF and LCM by Repeated Division

HCF: divide all the numbers by a prime that goes exactly into every one of them, and repeat. Stop when no prime goes into all of them. The HCF is the product of the primes you divided by.

LCM: divide by a prime that goes exactly into at least one of the numbers, copying down unchanged any number it does not go into. Repeat until every number has become 1. The LCM is the product of the primes you divided by.

Reading the question.

- "The largest number that divides A and B exactly" is the **HCF** of A and B.
- "The smallest positive number that is divisible by both A and B" is the **LCM** of A and B.
- "The smallest positive integer n such that An is a multiple of B": An is then a multiple of both A and B, so An is the LCM of A and B.
- **Remainders.** A number that leaves a remainder r when divided by A and by B is r more than a common multiple of A and B. The largest number that divides P and Q leaving a remainder r each time is the HCF of $P - r$ and $Q - r$ (it must be bigger than r).
- **Fractions.** Dividing the top and bottom of a fraction by their HCF gives the fraction in its simplest form. The LCM of two denominators is the lowest common denominator for adding or subtracting the fractions.

HCF or LCM?

- **HCF:** things are split into *equal* groups, packs or pieces with nothing left over, and the question asks for the *greatest* number of groups or the *greatest* size. The answer is a common *factor*, so it is no bigger than the numbers you start with.
- **LCM:** things happen again and again at regular intervals and the question asks when they next happen *together*, or equal totals have to be made from different sizes. The answer is a common *multiple*, so it is no smaller than the numbers you start with.
- **Clue words.** "Greatest" or "largest" with equal sharing points to the *highest* common factor; "next" or "again together" points to the *lowest* common multiple.
- **Counting.** When two things repeat every a and every b units and first meet again at the LCM, the number of each up to that point is LCM $\div\, a$ and LCM $\div\, b$.
- **Units.** Work out the LCM in the units given, then change it to the units the question asks for, such as seconds to minutes, or to a clock time.

Finding an unknown number from the HCF and LCM. Compare the numbers one prime at a time. In the **HCF**, each prime has the *smallest* index found among the numbers, so a prime that is missing from even one of the numbers is missing from the HCF. In the **LCM**, each prime has the *largest* index found among the numbers. For each prime in turn, decide which index the unknown

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number must have, or which indices it is allowed to have. It helps to set out a table with one column for each prime and one row each for the known numbers, the HCF and the LCM, then fill in the row for the unknown number.

Building Cuboids from Unit Cubes

- Each unit cube has a volume of 1 cubic unit, so the number of cubes used is the volume of the cuboid: length $\times$ breadth $\times$ height.
- Write the number of cubes as a product of its prime factors. The three edge lengths share out exactly these prime factors between them.
- Use the conditions in the question to decide how the factors are shared out: for example, the perimeter of the top is $2 \times (\text{length} + \text{breadth})$, a square top has equal length and breadth, and an edge may have to be longer than a given length.
- Try the possible ways one at a time, and check every one of them, so that you can be sure your answer is the only one that fits.

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Exercises

Use prime factorisation rather than a calculator unless a question says otherwise.

Section 1: Factors, Multiples and Prime Numbers

1. List all the prime numbers between 100 and 150.

2. State whether each number is prime or composite.
(a) 187 (b) 221

Section 2: Prime Factorisation and Index Notation

3. Write each number as a product of its prime factors, giving your answer in index notation.
(a) 1540 (b) 2541

4. When 810 is written as a product of its prime factors, $810 = 2 \times 3^x \times 5^y$. Find the values of x and y.

Section 3: Perfect Squares, Perfect Cubes and Their Roots

5. Use prime factorisation to find
(a) $\sqrt{1225}$ (b) $\sqrt{4356}$

6. (a) Find the smallest positive integer k such that $\sqrt[3]{4116k}$ is a whole number.
(b) Hence find the value of $\sqrt[3]{4116k}$.

7. $T = 3^7 \times 5^{11} \times 13^9$, and the number $N = T \times k$ is a perfect cube. Find the smallest positive integer value of k, giving your answer as a product of its prime factors.

8. $N = a^{10} \times b^{14} \times c^6$, where a, b and c are different prime numbers.
(a) Explain why N is a perfect square.
(b) Explain why N is not a perfect cube.

9. Given that $6000 = 2^4 \times 3 \times 5^3$, find the smallest positive integer
(a) k such that $6000k$ is a perfect square,
(b) m such that $6000m$ is a perfect cube,
(c) p such that $\dfrac{6000}{p}$ is a perfect cube.

10. It is given that $5808 = 2^4 \times 3 \times 11^2$. Find the smallest positive integer
(a) x such that $5808x$ is a perfect cube,
(b) y such that $5808y$ is both a perfect square and a perfect cube.
Leave both answers as products of their prime factors.

11. Given that $2^6 \times 21^2 = 28\,224$, find the value of $\sqrt{28\,224} \times \sqrt[3]{2\frac{10}{27}}$.

12. Find the difference between the first perfect cube that ends in the digit 3 and the first perfect square that ends in the digit 6.

13. Work out the following without using a calculator.
(a) $7^3 - \sqrt{2500} + \sqrt[3]{343}$ (b) $\sqrt{144} + \sqrt[3]{216} \times \sqrt{13^2 - 5^2} - \sqrt[3]{8}$

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14. Use a calculator to work out the following.

(a) $53^3 + \sqrt{7744} - 76^2 \times \sqrt[3]{4913}$ (b) $\sqrt{5776} \div \sqrt[3]{6859} + 27^2 + 13^3$

Section 4: Highest Common Factor and Lowest Common Multiple

15. Find the highest common factor (HCF) of each set of numbers.

(a) 90 and 225 (b) 42, 70 and 98

16. Find the lowest common multiple (LCM) of each set of numbers.

(a) 45 and 72 (b) 15, 42 and 98

17. Find the HCF and the LCM of each set of numbers.

(a) 36, 60 and 150 (b) 72, 120 and 300

18. Find the HCF and the LCM of each set of numbers, giving your answers as products of prime factors in index notation.

(a) $2^5 \times 3^2 \times 5^4$ and 6300 (b) $3^2 \times 5 \times 11^2$, $2 \times 3^3 \times 5^2 \times 11$ and $2^3 \times 3 \times 5 \times 13$

19. (a) Write each number as a product of its prime factors in index notation. (i) 420 (ii) 630

(b) Hence find

(i) the HCF of 420 and 630,

(ii) the LCM of 420 and 630.

(c) List all the common multiples of 420 and 630 that are less than 5000.

20. In prime factor form, $180 = 2^2 \times 3^2 \times 5$ and $294 = 2 \times 3 \times 7^2$. Find

(a) the largest number that divides both 180 and 294 exactly,

(b) the smallest positive integer n such that $180n$ is a multiple of 294,

(c) the smallest positive integer p such that $\dfrac{294}{p}$ is a perfect square.

21. Find the smallest whole numbers x and y such that $2^x \times 5^y \times 7^2$ is a multiple of 140.

22. In prime factor form, $360 = 2^3 \times 3^2 \times 5$ and $840 = 2^3 \times 3 \times 5 \times 7$.

(a) Find the largest integer that is a factor of both 360 and 840.

(b) Hence write the fraction $\dfrac{360}{840}$ in its simplest form.

23. (a) Write 1053 as a product of its prime factors.

(b) Given that $2925 = 3^2 \times 5^2 \times 13$, find the highest common factor of 1053 and 2925.

(c) Find the smallest positive integer k such that $\dfrac{1053}{k}$ is a perfect cube.

24. (a) Write 1188 as a product of its prime factors.

(b) In prime factor form, $1080 = 2^3 \times 3^3 \times 5$. Find the highest common factor of 1188 and 1080.

(c) Hence find the largest whole number that divides both 1193 and 1085, leaving a remainder of 5 each time.

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Section 5: HCF and LCM Word Problems

25. 125 adults and 182 children signed up for a beach clean-up. On the day, 5 of the adults and 2 of the children could not come. The people who came are split into as many teams as possible, so that every team has the same number of adults and the same number of children.

(a) What is the greatest number of teams?

(b) How many adults and how many children are there in each team?

26. Four jets in a musical fountain shoot water into the air every 1 minute, every 1 minute 15 seconds, every 1 minute 24 seconds and every 1 minute 40 seconds respectively. All four jets shot water together at 07 45. At what time will all four jets next shoot water together?

27. Three bells ring at regular intervals of 50 minutes, 75 minutes and 90 minutes. They ring together at the start of a festival.

(a) How long after the start will all three bells next ring together? Give your answer in hours and minutes.

(b) How many times does the 50-minute bell ring in that time, not counting its ring at the start?

28. Two gear wheels turn together. Wheel A has 48 teeth and wheel B has 60 teeth. As the wheels turn, one tooth of B passes the point where the wheels meet for every tooth of A that passes it. One tooth on each wheel is marked, and the two marked teeth are touching at the start.

(a) How many teeth of wheel A pass the meeting point before the two marked teeth touch again?

(b) How many complete turns does each wheel make in that time?

Section 6: Finding an Unknown Number from the HCF and LCM

29. The HCF and LCM of three numbers are 12 and $2^3 \times 3^2 \times 5 \times 7 \times 11$ respectively. Two of the numbers are 180 and 252. Find the largest possible value of the third number, leaving your answer as a product of prime factors in index notation.

30. n is a common multiple of 6 and 8, and n is also a factor of 144. Find all the possible values of n.

31. $M = 2^a \times 3^4 \times 5^b$ and $N = 2^3 \times 3^c \times 5$, where a, b and c are positive integers. The HCF of M and N is $2^2 \times 3^2 \times 5$ and the LCM of M and N is $2^3 \times 3^4 \times 5^3$. Find the values of a, b and c.

32. In prime factor form, $2340 = 2^2 \times 3^2 \times 5 \times 13$.

(a) Write 130 as a product of its prime factors.

(b) The LCM of 130 and a positive integer R is 2340. Find all the possible values of R.

Section 7: Building Cuboids from Unit Cubes

33. Siti uses all 240 of her unit cubes to make a cuboid. Every edge of the cuboid is made up of more than 4 cubes. Find the number of cubes along each edge of the cuboid.

34. Peter uses all of his 252 one-centimetre cubes to build a cuboid whose top is a square. Every edge of the cuboid is longer than 1 cm, and the height of the cuboid is less than 10 cm. Find the dimensions of the cuboid.

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Solutions to Exercises

Section 1 — Factors, Multiples and Prime Numbers (Questions 1–2)

1 $\sqrt{150} = 12.25$ (2 d.p.), so every composite number up to 150 can be divided by 2, 3, 5, 7 or 11. Crossing out the multiples of $2, 3, 5, 7$ and 11 between 100 and 150 leaves

$$\boxed{101, 103, 107, 109, 113,\ 127, 131, 137, 139, 149}$$

2(a) $187 = 11 \times 17$, so 187 is $\boxed{\text{composite}}$.

2(b) $221 = 13 \times 17$, so 221 is $\boxed{\text{composite}}$.

Section 2 — Prime Factorisation and Index Notation (Questions 3–4)

3

(a)

2	1540
2	770
5	385
7	77
11	11
	1

$\boxed{2^2 \times 5 \times 7 \times 11}$

(b)

3	2541
7	847
11	121
11	11
	1

$\boxed{3 \times 7 \times 11^2}$

4 $810 = 2 \times 3 \times 3 \times 3 \times 3 \times 5 = 2 \times 3^4 \times 5$ (divide by 2 once, by 3 four times, then by 5 once), so $\boxed{x = 4,\ y = 1}$

Section 3 — Perfect Squares, Perfect Cubes and Their Roots (Questions 5–14)

5(a) $1225 = 5^2 \times 7^2$; $\sqrt{1225} = 5 \times 7 = \boxed{35}$

5(b) $4356 = 2^2 \times 3^2 \times 11^2$; $\sqrt{4356} = 2 \times 3 \times 11 = \boxed{66}$

6(a) $\sqrt[3]{4116k}$ is a whole number when $4116k$ is a perfect cube.
$4116 = 2^2 \times 3 \times 7^3$; $4116k = 2^3 \times 3^3 \times 7^3$
$k = 2 \times 3^2 = \boxed{18}$

6(b) $\sqrt[3]{4116k} = \sqrt[3]{2^3 \times 3^3 \times 7^3} = 2 \times 3 \times 7 = \boxed{42}$

7 Raise each index to a multiple of 3:
$T \times k = 3^9 \times 5^{12} \times 13^9$
$k = \boxed{3^2 \times 5}$

8(a) Every index (10, 14 and 6) is even, so $N = \left(a^5 \times b^7 \times c^3\right)^2$ is $\boxed{\text{a perfect square}}$.

8(b) The indices 10 and 14 are not multiples of 3, so the prime factors of N cannot be shared out into three identical groups: N is $\boxed{\text{not a perfect cube}}$.

9(a) $6000k = 2^4 \times 3^2 \times 5^4$; $k = 3 \times 5 = \boxed{15}$

9(b) $6000m = 2^6 \times 3^3 \times 5^3$; $m = 2^2 \times 3^2 = \boxed{36}$

9(c) $\dfrac{6000}{p} = 2^3 \times 5^3$; $p = 2 \times 3 = \boxed{6}$

10(a) $5808x = 2^6 \times 3^3 \times 11^3$; $x = \boxed{2^2 \times 3^2 \times 11}$

10(b) Every index must be a multiple of 6:
$5808y = 2^6 \times 3^6 \times 11^6$
$y = \boxed{2^2 \times 3^5 \times 11^4}$

11 $\sqrt{28\,224} = \sqrt{2^6 \times 21^2} = 2^3 \times 21 = 168$
$\sqrt[3]{2\frac{10}{27}} = \sqrt[3]{\dfrac{64}{27}} = \dfrac{\sqrt[3]{2^6}}{\sqrt[3]{3^3}} = \dfrac{4}{3}$
$168 \times \dfrac{4}{3} = \boxed{224}$

12 Perfect cubes: $1, 8, 27, 64, 125, 216, 343, \ldots$; the first that ends in 3 is 343.
Perfect squares: $1, 4, 9, 16, \ldots$; the first that ends in 6 is 16.
Difference $= 343 - 16 = \boxed{327}$

13(a) $343 - 50 + 7 = \boxed{300}$

13(b) $\sqrt{13^2 - 5^2} = \sqrt{169 - 25} = \sqrt{144} = 12$
$12 + 6 \times 12 - 2 = 12 + 72 - 2 = \boxed{82}$

14(a) $148\,877 + 88 - 5776 \times 17 = 148\,965 - 98\,192 = \boxed{50\,773}$

14(b) $76 \div 19 + 729 + 2197 = 4 + 2926 = \boxed{2930}$

Section 4 — Highest Common Factor and Lowest Common Multiple (Questions 15–24)

15(a) $90 = 2 \times 3^2 \times 5$, $225 = 3^2 \times 5^2$
HCF $= 3^2 \times 5 = \boxed{45}$

15(b) $42 = 2 \times 3 \times 7$, $70 = 2 \times 5 \times 7$, $98 = 2 \times 7^2$
HCF $= 2 \times 7 = \boxed{14}$

16(a) $45 = 3^2 \times 5$, $72 = 2^3 \times 3^2$
LCM $= 2^3 \times 3^2 \times 5 = \boxed{360}$

16(b) $15 = 3 \times 5$, $42 = 2 \times 3 \times 7$, $98 = 2 \times 7^2$
LCM $= 2 \times 3 \times 5 \times 7^2 = \boxed{1470}$

17(a) $36 = 2^2 \times 3^2$, $60 = 2^2 \times 3 \times 5$, $150 = 2 \times 3 \times 5^2$
HCF $= 2 \times 3 = \boxed{6}$; LCM $= 2^2 \times 3^2 \times 5^2 = \boxed{900}$

17(b) $72 = 2^3 \times 3^2$, $120 = 2^3 \times 3 \times 5$, $300 = 2^2 \times 3 \times 5^2$
HCF $= 2^2 \times 3 = \boxed{12}$; LCM $= 2^3 \times 3^2 \times 5^2 = \boxed{1800}$

18(a) $6300 = 2^2 \times 3^2 \times 5^2 \times 7$
HCF $= \boxed{2^2 \times 3^2 \times 5^2}$; LCM $= \boxed{2^5 \times 3^2 \times 5^4 \times 7}$

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18(b) HCF = $\boxed{3 \times 5}$;
LCM = $\boxed{2^3 \times 3^3 \times 5^2 \times 11^2 \times 13}$

19(a) (i) $420 = \boxed{2^2 \times 3 \times 5 \times 7}$
(ii) $630 = \boxed{2 \times 3^2 \times 5 \times 7}$

19(b) (i) HCF $= 2 \times 3 \times 5 \times 7 = \boxed{210}$
(ii) LCM $= 2^2 \times 3^2 \times 5 \times 7 = \boxed{1260}$

19(c) The common multiples are the multiples of the LCM, 1260: 1260, 2520, 3780, 5040, … Those less than 5000 are
$\boxed{1260,\ 2520,\ 3780}$

20(a) The largest number dividing both exactly is the HCF: $2 \times 3 = \boxed{6}$

20(b) LCM of 180 and $294 = 2^2 \times 3^2 \times 5 \times 7^2$;
$180n = \text{LCM}$
$n = \dfrac{2^2 \times 3^2 \times 5 \times 7^2}{2^2 \times 3^2 \times 5} = 7^2 = \boxed{49}$

20(c) $\dfrac{294}{p} = 7^2$; $p = 2 \times 3 = \boxed{6}$

21 $140 = 2^2 \times 5 \times 7$. 7^2 already supplies the 7, so two factors of 2 and one factor of 5 are needed.
$\boxed{x = 2,\ y = 1}$

22(a) The largest integer that is a factor of both is the HCF:
$2^3 \times 3 \times 5 = \boxed{120}$

22(b) Divide the top and bottom by the HCF, 120:
$\dfrac{360}{840} = \dfrac{360 \div 120}{840 \div 120} = \boxed{\dfrac{3}{7}}$

23(a) $1053 = \boxed{3^4 \times 13}$

23(b) HCF $= 3^2 \times 13 = \boxed{117}$

23(c) $\dfrac{1053}{k} = 3^3$; $k = 3 \times 13 = \boxed{39}$

24(a) $1188 = \boxed{2^2 \times 3^3 \times 11}$

24(b) HCF $= 2^2 \times 3^3 = \boxed{108}$

24(c) Take away the remainder: $1193 - 5 = 1188$ and $1085 - 5 = 1080$. The number divides both 1188 and 1080 exactly, so the largest such number is their HCF, which is bigger than 5.
$\boxed{108}$

Section 5 — HCF and LCM Word Problems (Questions 25–28)

25(a) $125 - 5 = 120$ adults and $182 - 2 = 180$ children came. The number of teams must divide both exactly and be as large as possible, so it is the HCF.
$120 = 2^3 \times 3 \times 5$, $180 = 2^2 \times 3^2 \times 5$
HCF $= 2^2 \times 3 \times 5 = \boxed{60 \text{ teams}}$

25(b) $120 \div 60 = 2$; $180 \div 60 = 3$
$\boxed{2 \text{ adults and } 3 \text{ children}}$

26 The intervals are 60, 75, 84 and 100 seconds. The jets next shoot water together after the LCM.
$60 = 2^2 \times 3 \times 5$, $75 = 3 \times 5^2$, $84 = 2^2 \times 3 \times 7$,
$100 = 2^2 \times 5^2$
LCM $= 2^2 \times 3 \times 5^2 \times 7 = 2100$ s
2100 s = 35 min
07 45 + 35 min = $\boxed{08\,20}$

27(a) $50 = 2 \times 5^2$, $75 = 3 \times 5^2$, $90 = 2 \times 3^2 \times 5$
LCM $= 2 \times 3^2 \times 5^2 = 450$ min
450 min = $\boxed{7 \text{ h } 30 \text{ min}}$

27(b) $450 \div 50 = \boxed{9 \text{ times}}$

28(a) The marked tooth of A is back at the meeting point after every 48 teeth, and the marked tooth of B after every 60 teeth. So the number of teeth that pass is a common multiple of 48 and 60; the first is the LCM.
$48 = 2^4 \times 3$, $60 = 2^2 \times 3 \times 5$
LCM $= 2^4 \times 3 \times 5 = \boxed{240 \text{ teeth}}$

28(b) A: $240 \div 48 = 5$; B: $240 \div 60 = 4$
$\boxed{A:\ 5 \text{ turns};\ B:\ 4 \text{ turns}}$

Section 6 — Finding an Unknown Number from the HCF and LCM (Questions 29–32)

29 $180 = 2^2 \times 3^2 \times 5$, $252 = 2^2 \times 3^2 \times 7$, $12 = 2^2 \times 3$,
LCM $= 2^3 \times 3^2 \times 5 \times 7 \times 11$. For the third number:
2: the LCM has 2^3, but 180 and 252 have only 2^2, so it must have 2^3.
3: the HCF has only 3^1, while 180 and 252 have 3^2, so exactly one 3.
5, 7: already supplied by 180 and 252, so each may or may not be there; for the largest value, include both.
11: neither number has it, so it must be there.
Largest third number = $\boxed{2^3 \times 3 \times 5 \times 7 \times 11}$

30 n is a common multiple of 6 and 8, so it is a multiple of their LCM, $24 = 2^3 \times 3$. The factors of $144 = 2^4 \times 3^2$ that are multiples of 24 are
$\boxed{24,\ 48,\ 72,\ 144}$

31 Compare one prime at a time.
2: the HCF has 2^2 and N has 2^3, so M has 2^2: $a = 2$.
3: the HCF has 3^2 and M has 3^4, so N has 3^2: $c = 2$.
5: the LCM has 5^3 and N has 5^1, so M has 5^3: $b = 3$.
$\boxed{a = 2,\ b = 3,\ c = 2}$

32(a) $130 = \boxed{2 \times 5 \times 13}$

32(b) R must contain 2^2 and 3^2, which 130 does not supply; it may or may not contain 5 and 13, and it can have no other prime factor.
$R = 2^2 \times 3^2 = 36$, $2^2 \times 3^2 \times 5 = 180$, $2^2 \times 3^2 \times 13 = 468$ or $2^2 \times 3^2 \times 5 \times 13 = 2340$
$\boxed{36,\ 180,\ 468,\ 2340}$

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Section 7 — Building Cuboids from Unit Cubes (Questions 33–34)

33 $240 = 2^4 \times 3 \times 5 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$. Every edge has at least 5 cubes. If 5 is joined to another factor, that edge has at least 10 cubes, and the other two edges multiply to at most $240 \div 10 = 24$, which is less than 5×5: impossible. So 5 is an edge on its own, and $2, 2, 2, 2, 3$ must make two edges of at least 5: only $2 \times 3 = 6$ and $2 \times 2 \times 2 = 8$ work.

$\boxed{5 \text{ by } 6 \text{ by } 8 \text{ cubes}}$

34 $252 = 2^2 \times 3^2 \times 7$. The top is x cm by x cm, so x^2 is a factor of 252, with $x > 1$: $x = 2$, 3 or 6.
Heights: $252 \div 4 = 63$, $252 \div 9 = 28$, $252 \div 36 = 7$.
Only 7 is less than 10.

$\boxed{6 \text{ cm by } 6 \text{ cm by } 7 \text{ cm}}$

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